Resource-Aware Session Types for Digital Contracts

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Digital Contracts

- Programs to digitally facilitate the execution of a transaction between distrusting parties
- Transactions are carried out by miners and stored on a global distributed ledger, or blockchain
- User pays for the execution cost of transaction

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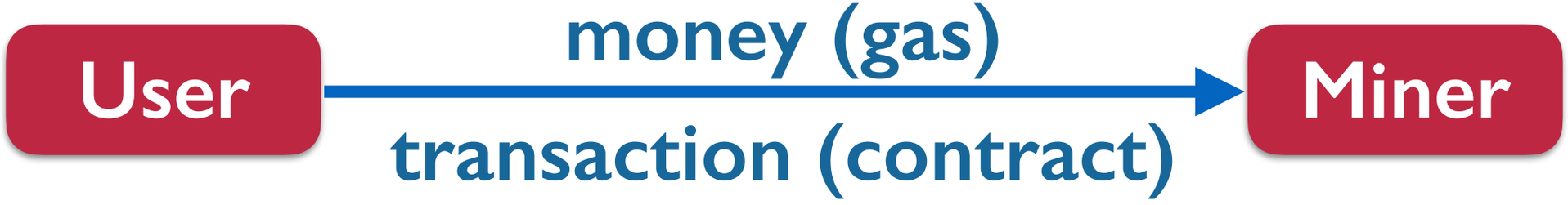
set standard assigns cost to each operation

Execution Model

Execution Model



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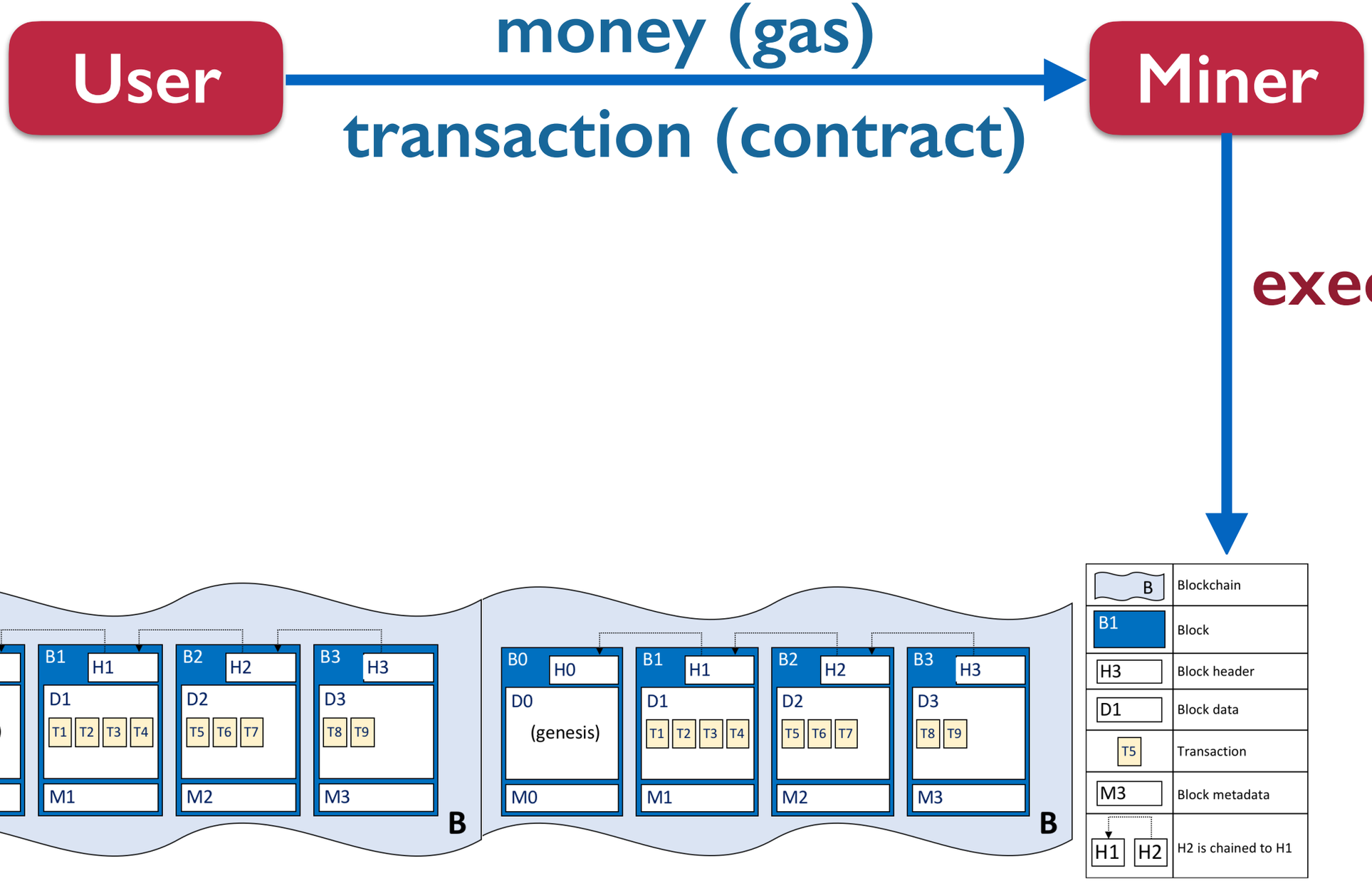
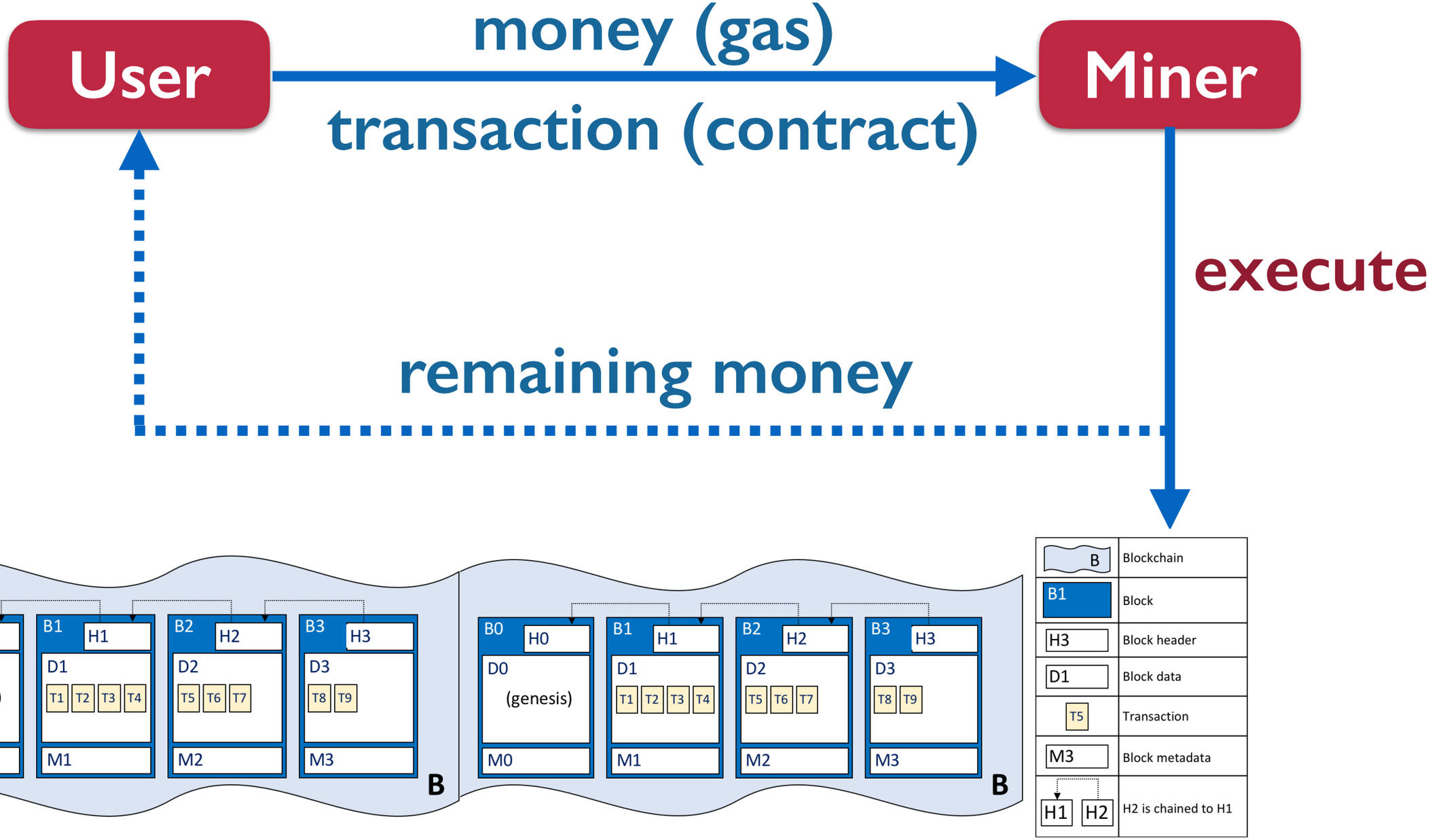


global ledger (blockchain)

new block

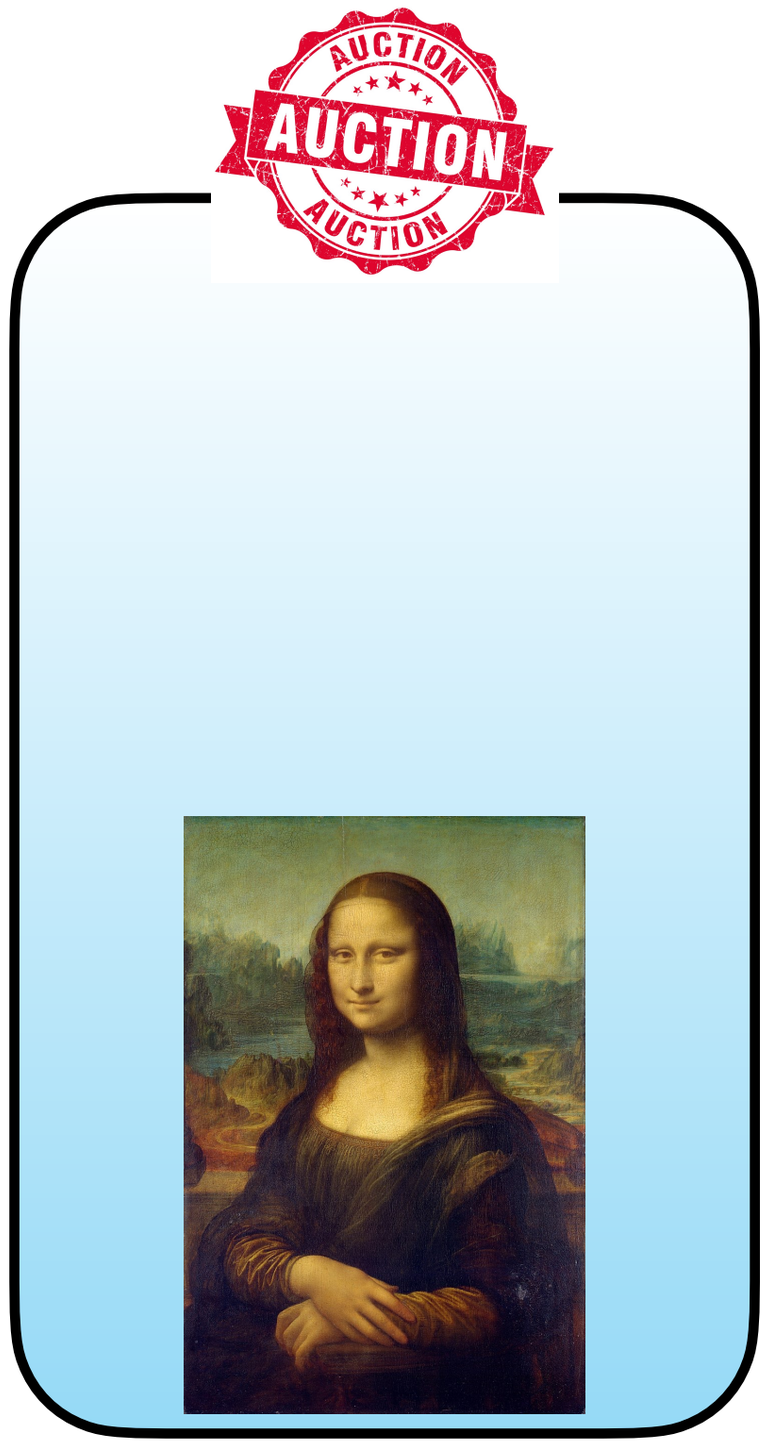
Execution Model



global ledger (blockchain)

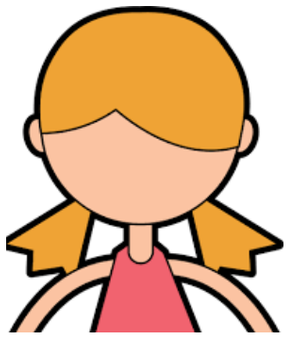
new block

Auction Contract



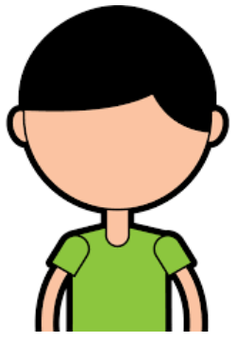
status: running

Auction Contract



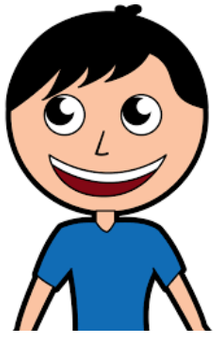
Bid 1

Bidder 1



Bid 2

Bidder 2

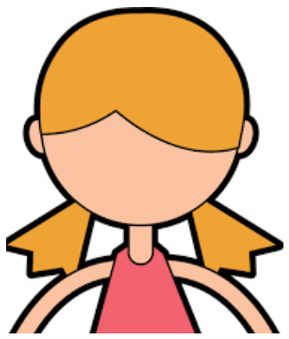


Bid 3

Bidder 3

status: running

Auction Contract



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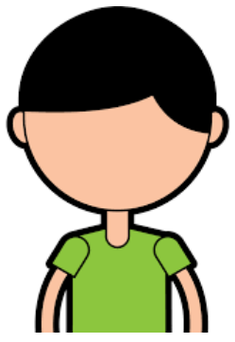
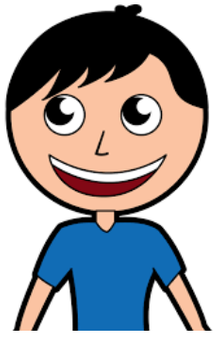


Bidder 2



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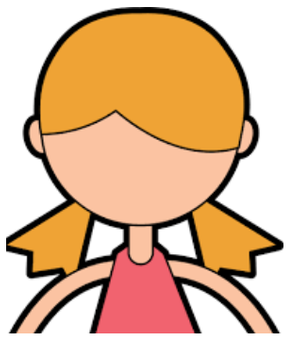

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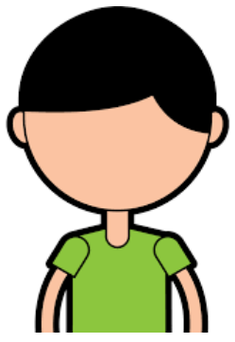
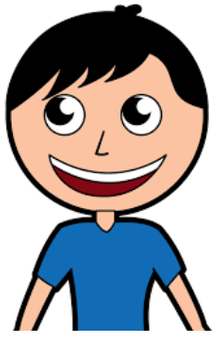


Bidder 2



Bidder 3

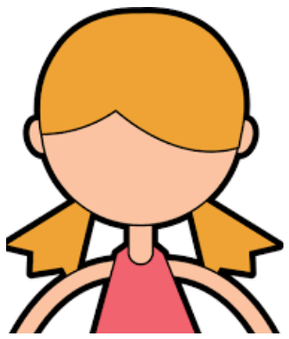
Bid 1

Bid 2

Bid 3

status: ended

Auction Contract



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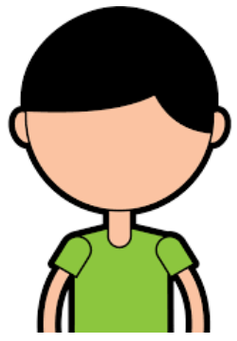
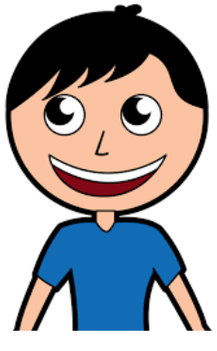


Bidder 2



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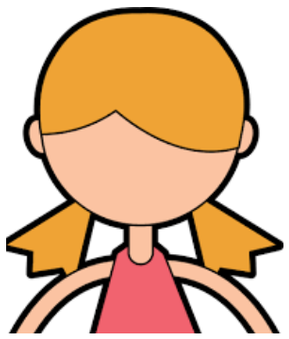

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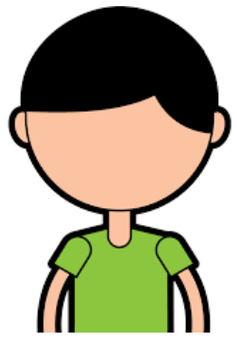

Bidder 2

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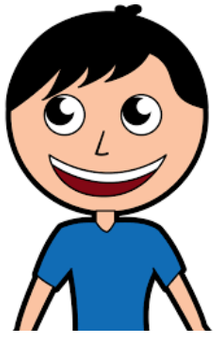

Bidder 3

Bid 3

status: ended

Auction in Solidity


```
function bid() public payable {
  bid = msg.value;
  bidder = msg.sender;
  pendingReturns[bidder] = bid;
  if (bid > highestBid) {
    highestBidder = bidder;
    highestBid = bid;
  }
}

function collect() public returns (bool) {
  require (msg.sender != highestBidder);
  uint amount = pendingReturns[msg.sender];
  msg.sender.send(amount);
  return true;
}
```

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Protocol is not statically enforced!

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set pendingReturns[msg.sender] = 0

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Linearity is not enforced!

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'send' transfers control to user who can call collect

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'send' should be the last instruction

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Re-entrancy Attack

'send' transfers control to user who can call collect

'send' should be the last instruction

Reentrancy Attacks in News

The DAO Attacked: Code Issue Leads to $60 Million Ether Theft

Michael del Castillo

Jun 17, 2016 at 14:00 UTC • Updated Jun 18, 2016 at 14:46 UTC

NEWS

ETHEREUM

ChainSecurity: Ethereum's Constantinople upgrade "enables new Reentrancy Attack"

JANUARY 15, 2019, 3:12PM EDT

Clever Ethereum honeypot lets coins come in but won't let them back out

John Biggs @johnbiggs / 1 year ago

Comment

Auction in Solidity


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Resource consumption?
User needs to pay appropriate gas

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Resource consumption?
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APPENDIX G. FEE SCHEDULE

The fee schedule G is a tuple of 31 scalar values corresponding to the relative costs, in gas, of a number of abstract operations that a transaction may effect.

Name	Value	Description*
G_{zero}	0	Nothing paid for operations of the set W_{zero}.
G_{base}	2	Amount of gas to pay for operations of the set W_{base}.
$G_{verylow}$	3	Amount of gas to pay for operations of the set $W_{verylow}$.
G_{low}	5	Amount of gas to pay for operations of the set W_{low}.
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$G_{extcode}$	700	Amount of gas to pay for operations of the set $W_{extcode}$.

EVM cost model

Auction in Solidity


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}
```

Automatic Resource Analysis

Resource consumption?

User needs to pay appropriate gas

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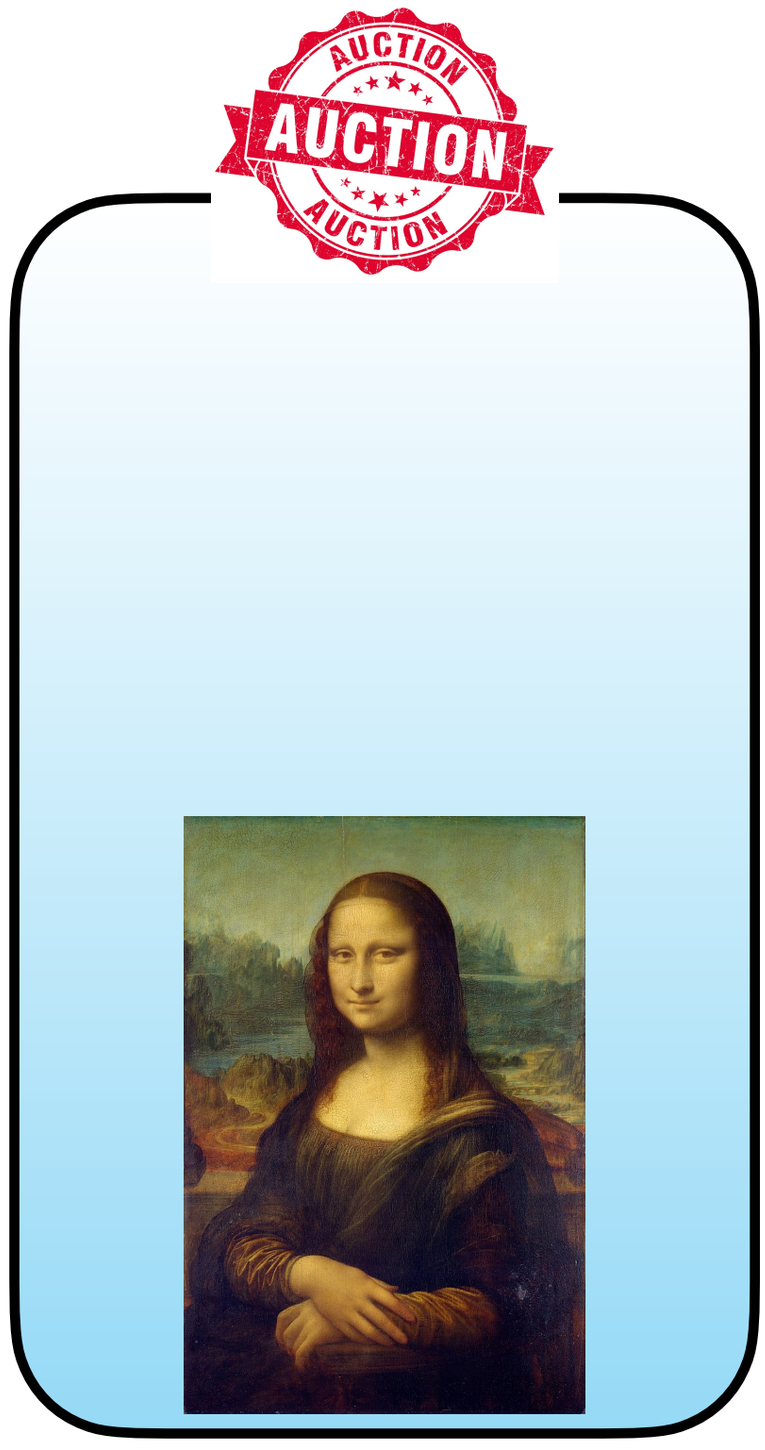
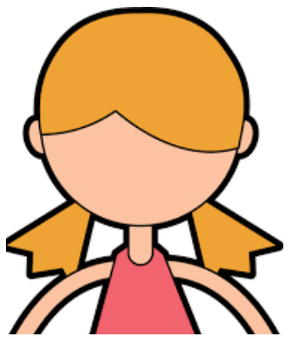
EVM cost model

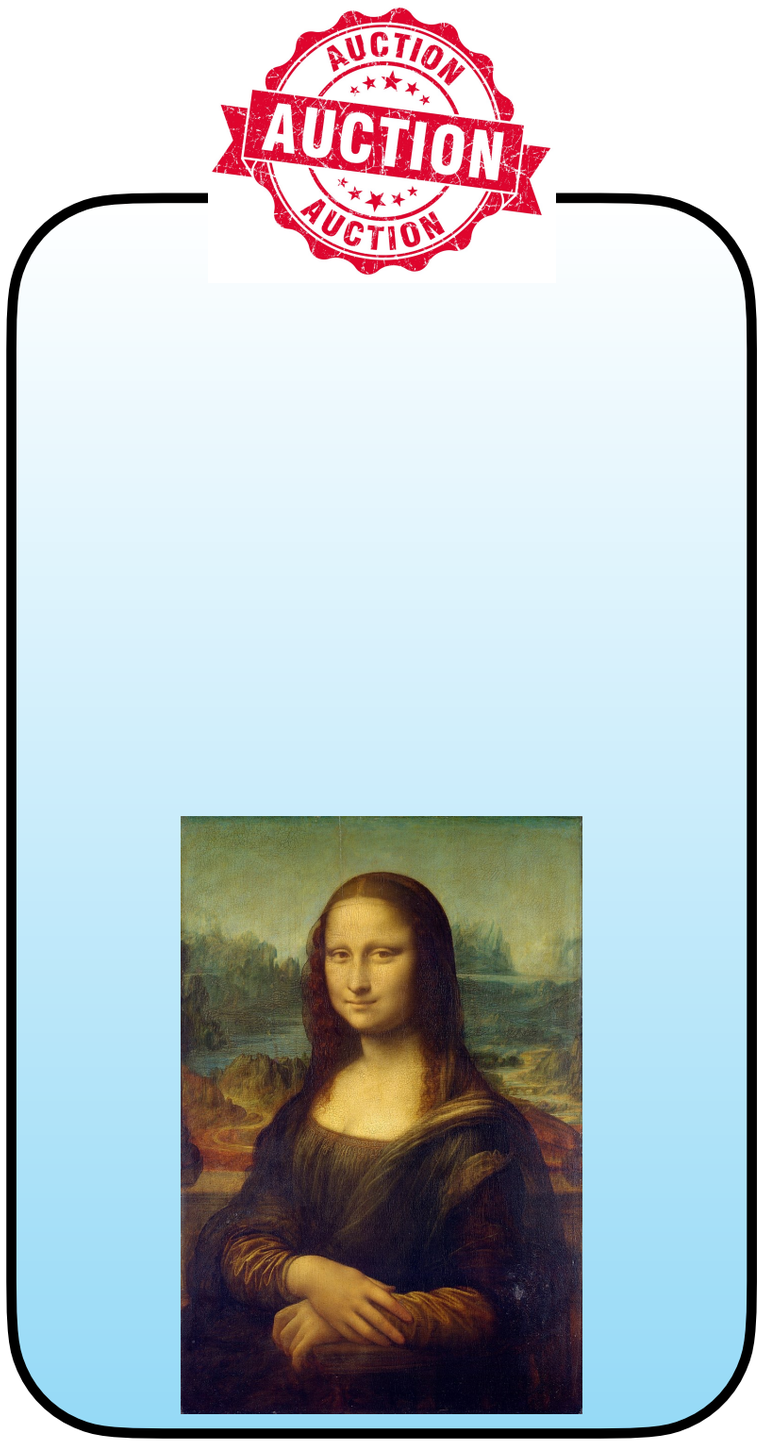
Auction Protocol



Auction Protocol

bidding phase





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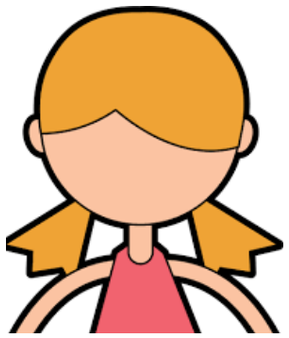
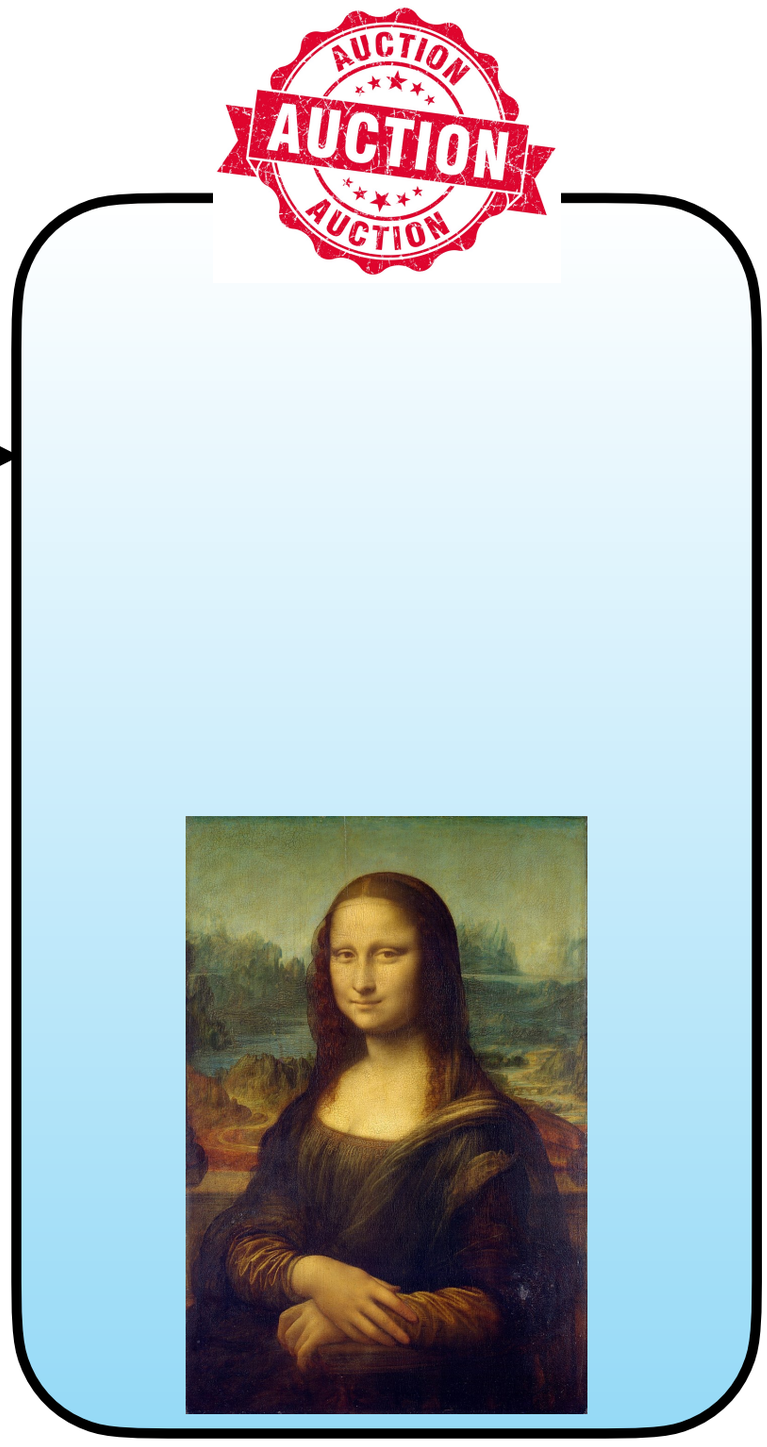


id, money



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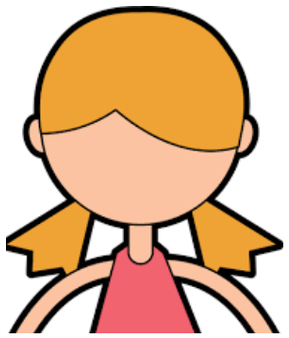
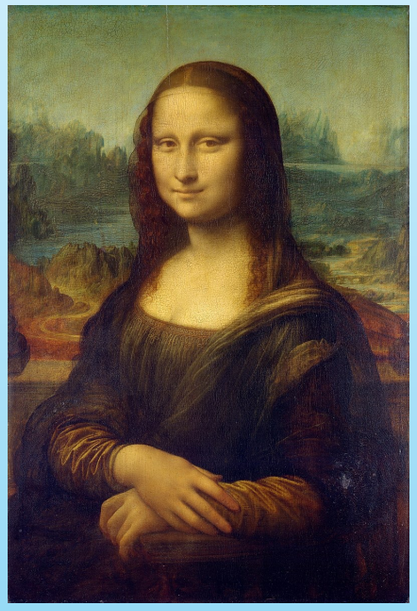


id, money

recurse



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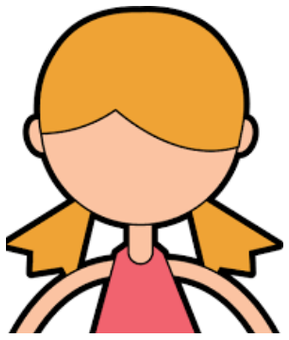

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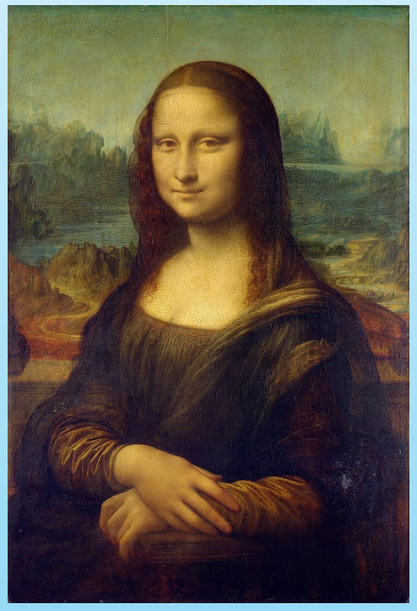
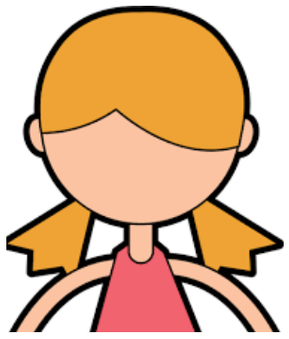
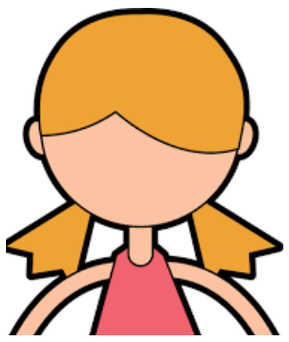


ended phase



Auction Protocol

bidding phase



id, money

recurse

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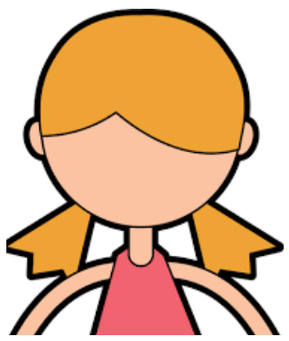

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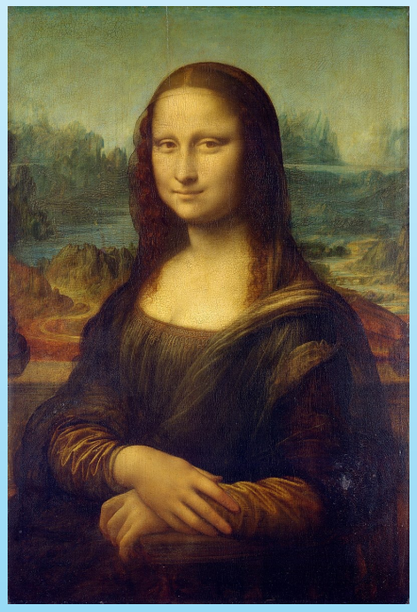

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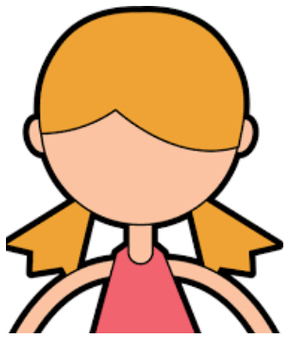
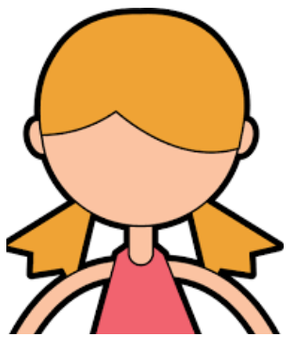


id, money

recurse

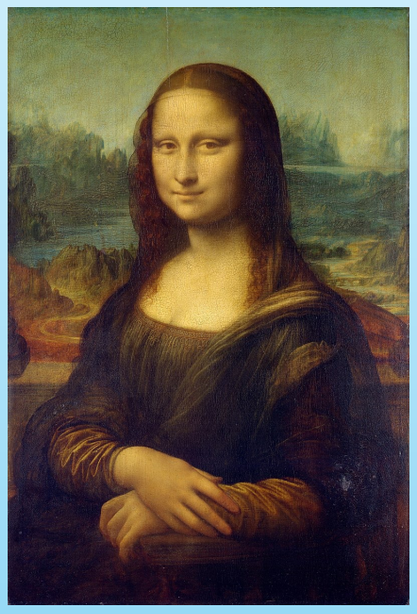
ended phase



id

recurse

monalisa / money



Auction as a Session Type

$$\begin{aligned}\text{auction} = \oplus\{&\mathbf{running} : \&\{\mathbf{bid} : \text{id} \to \text{money} \multimap \text{auction}\},\\ &\mathbf{ended} : \&\{\mathbf{collect} : \text{id} \to \oplus\{\mathbf{won} : \text{monalisa} \otimes \text{auction},\\ &\qquad\mathbf{lost} : \text{money} \otimes \text{auction}\}\}\}\end{aligned}$$

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Auction is the Provider | Bidder is the Client

Auction as a Session Type

sends status of auction

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Auction is the Provider | Bidder is the Client

Auction as a Session Type

sends status of auction

offers choice of bidding

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Auction is the Provider | Bidder is the Client

Auction as a Session Type

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Talk Outline

Talk Outline

Session Types

Talk Outline

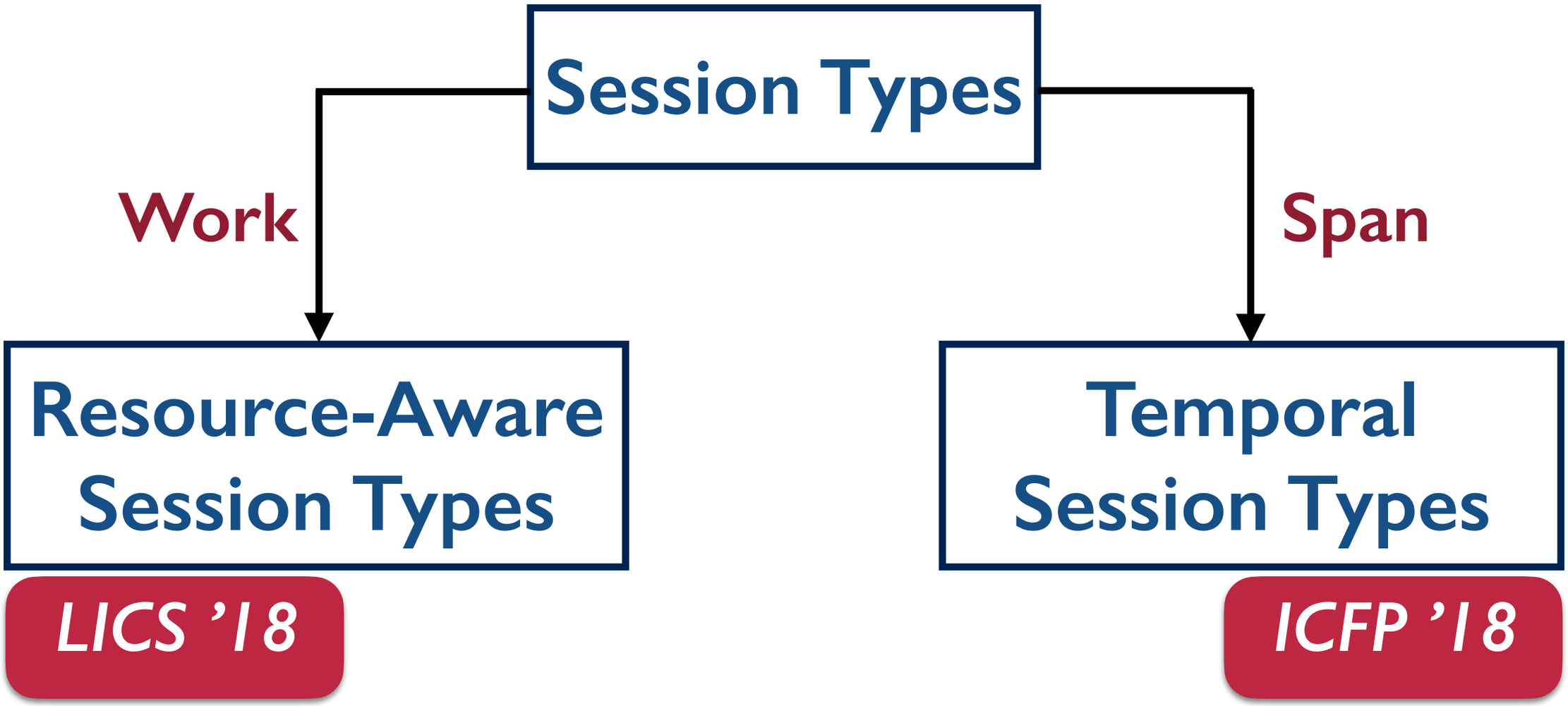
Session Types

Work

Resource-Aware Session Types

LICS '18

Talk Outline



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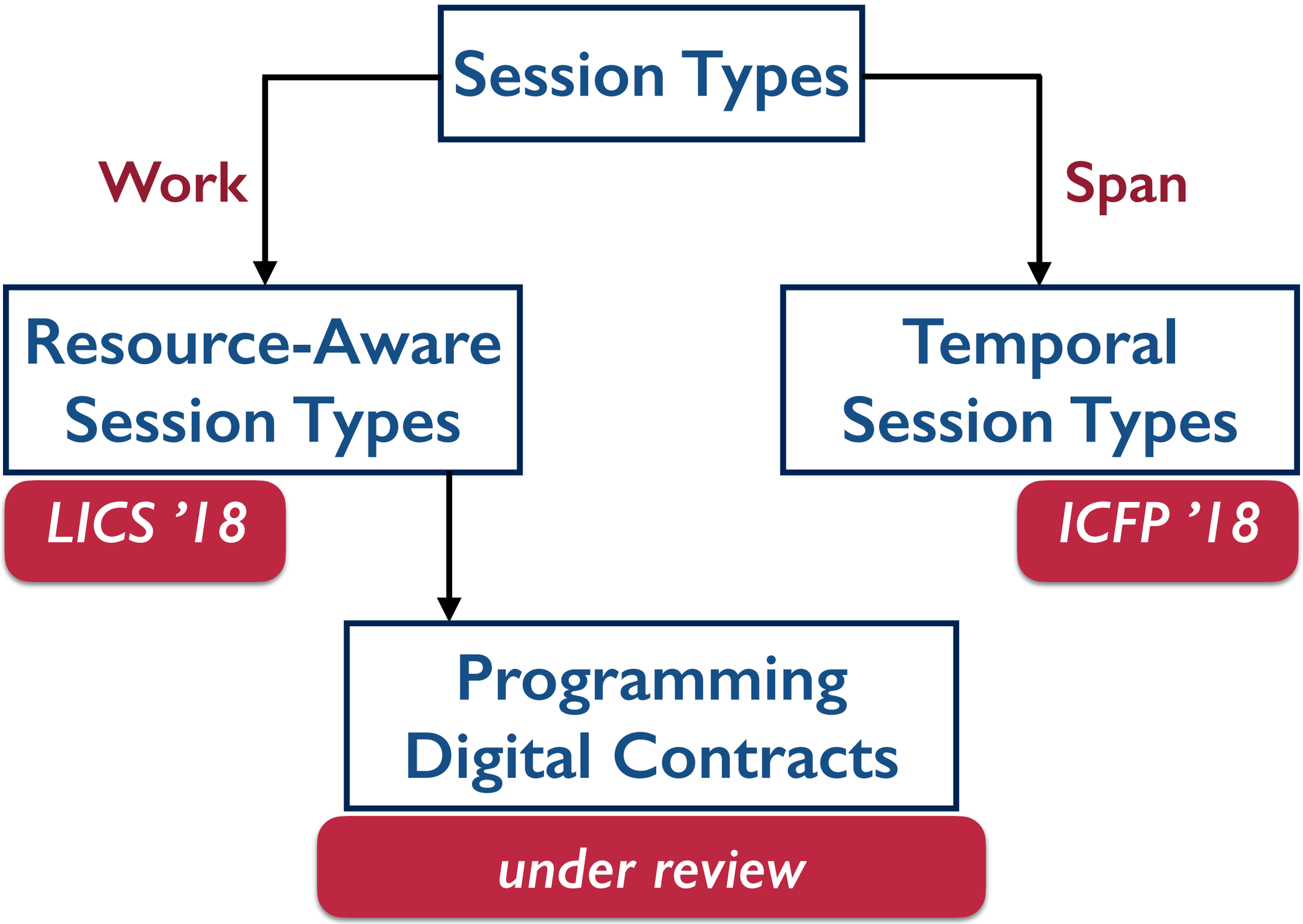

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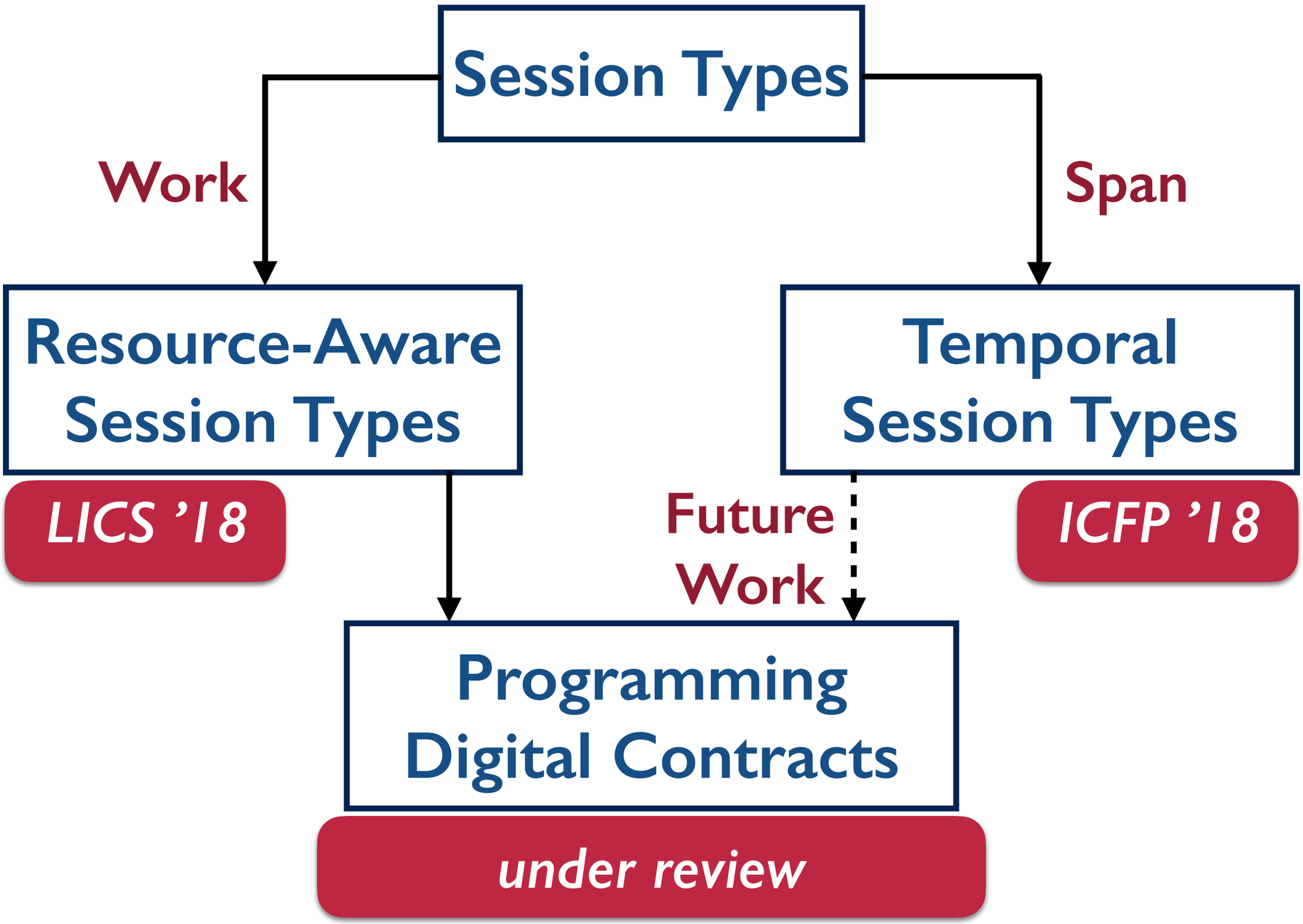

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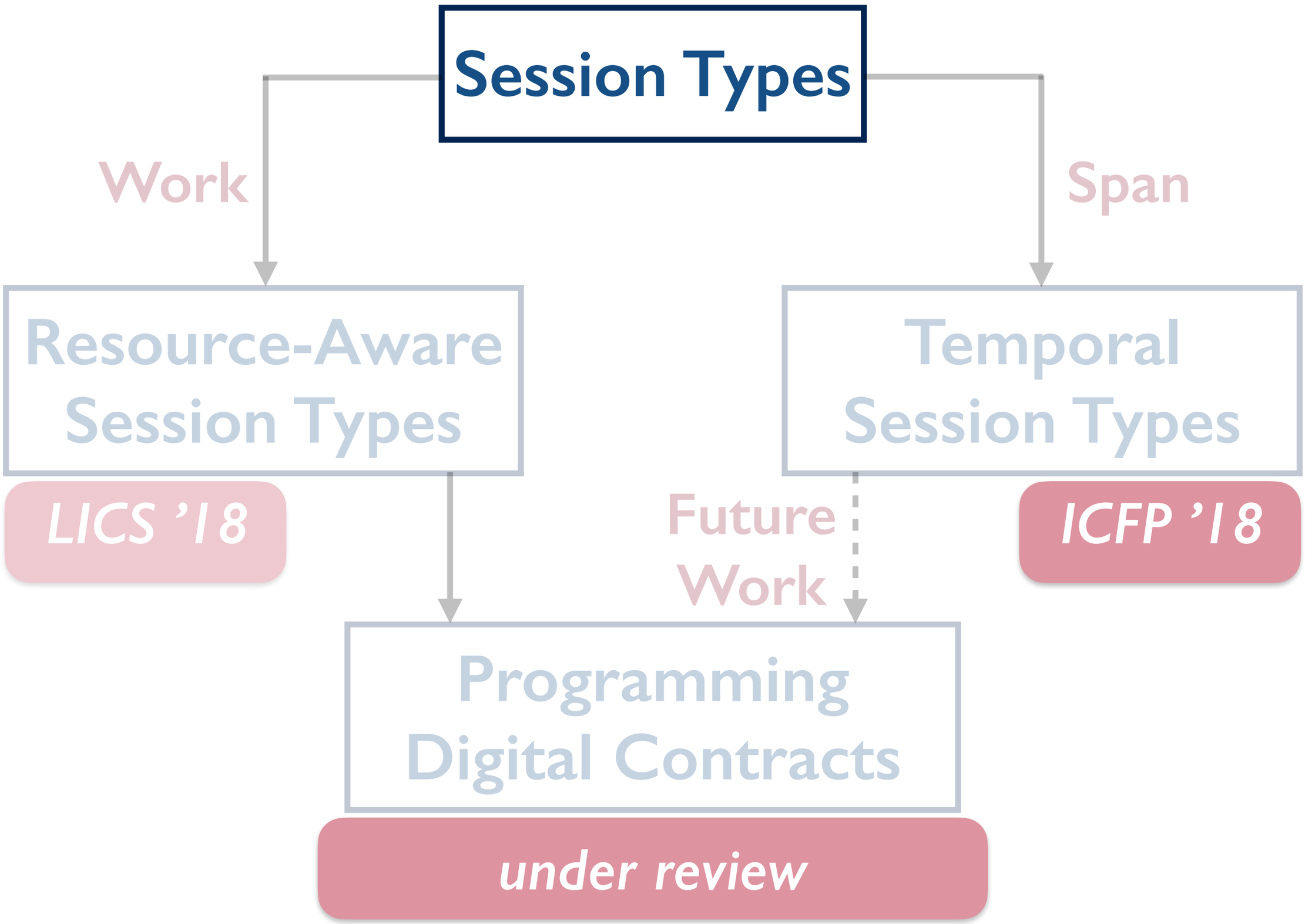

What are Session Types?

- Implement message-passing concurrent programs
- Communication via typed bi-directional channels
- Communication protocol enforced by session types

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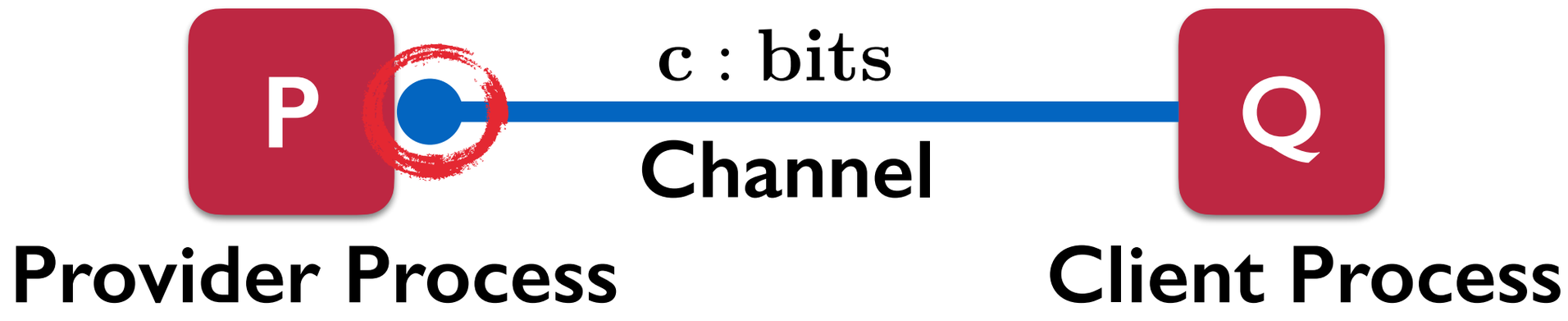

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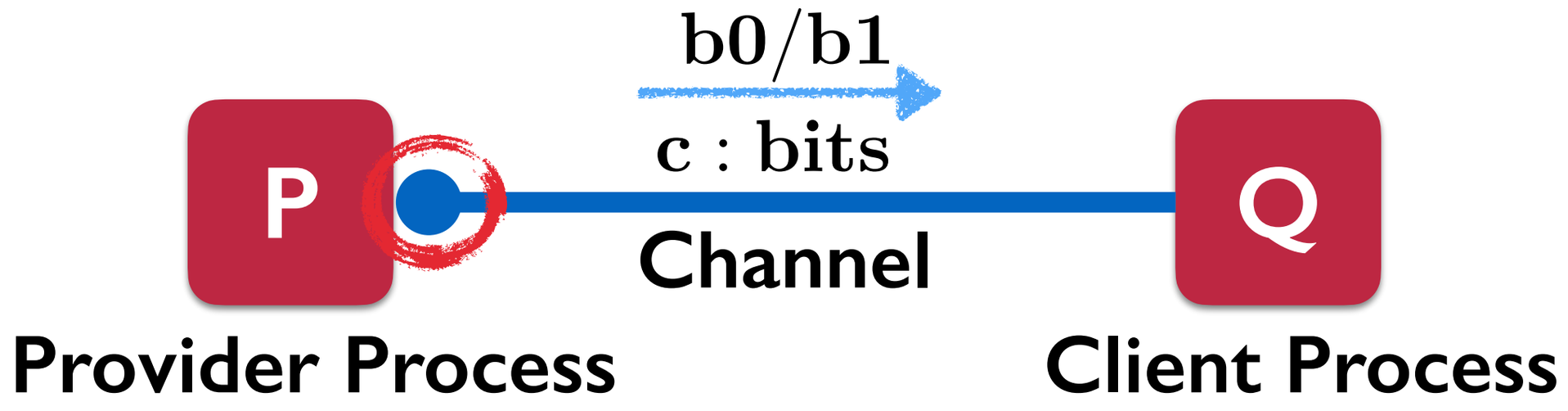

Example: Queues

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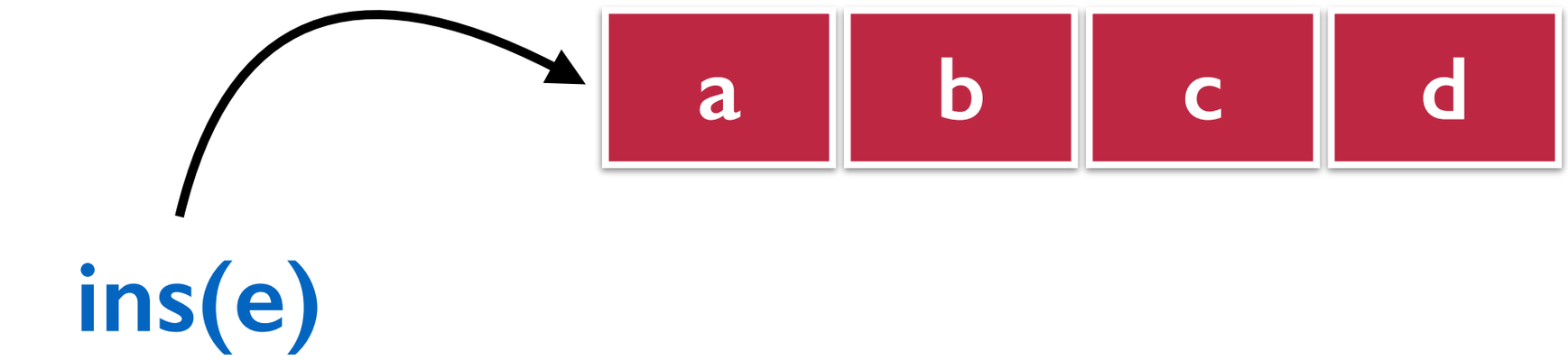

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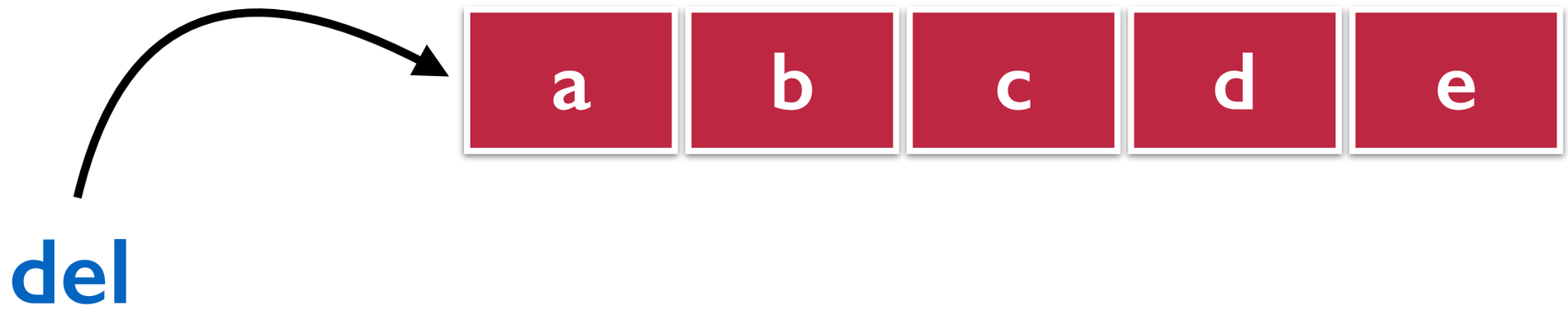

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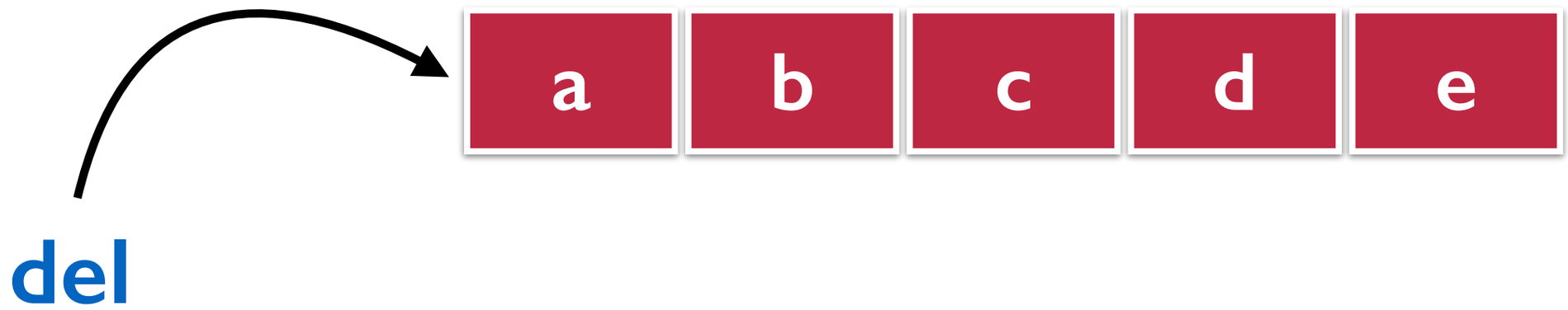

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terminate

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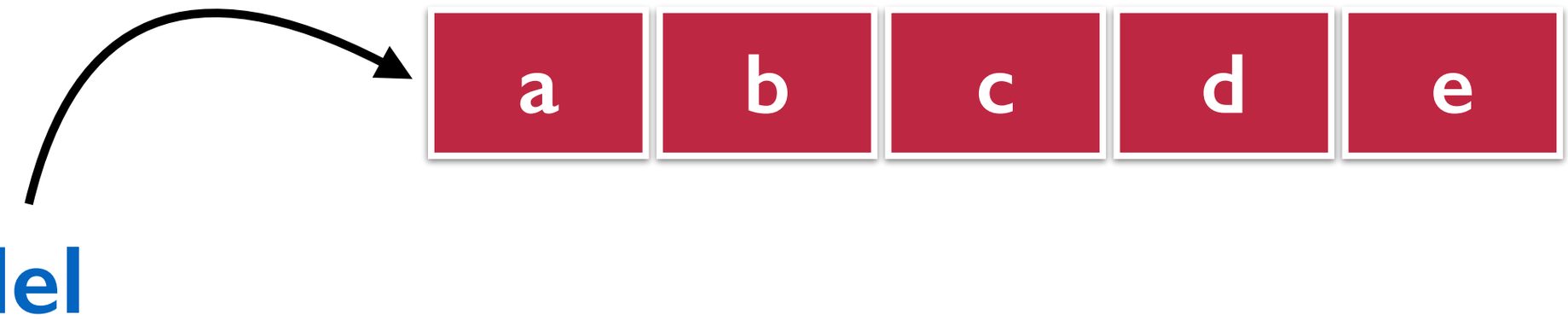
Example: Queues



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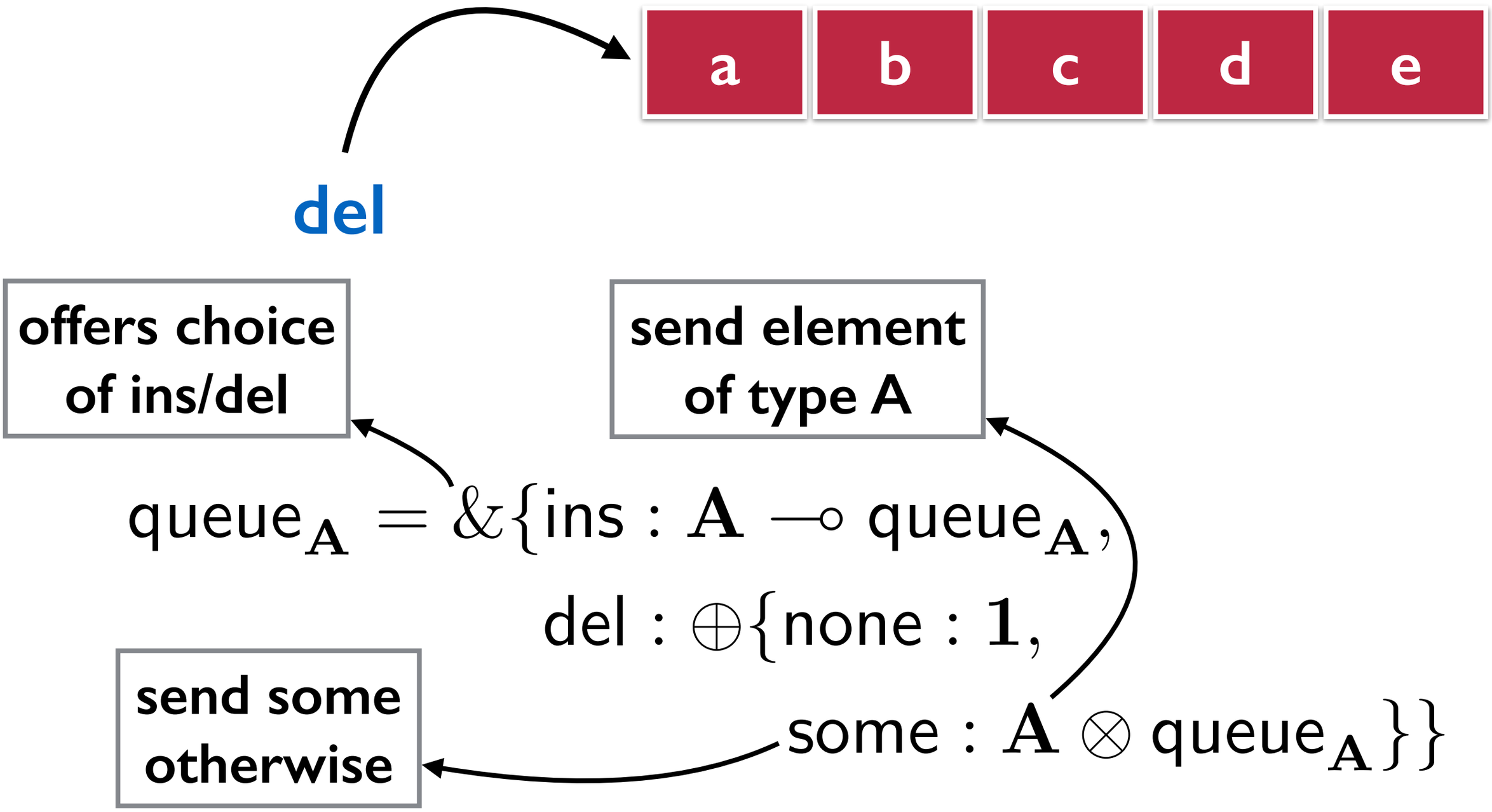

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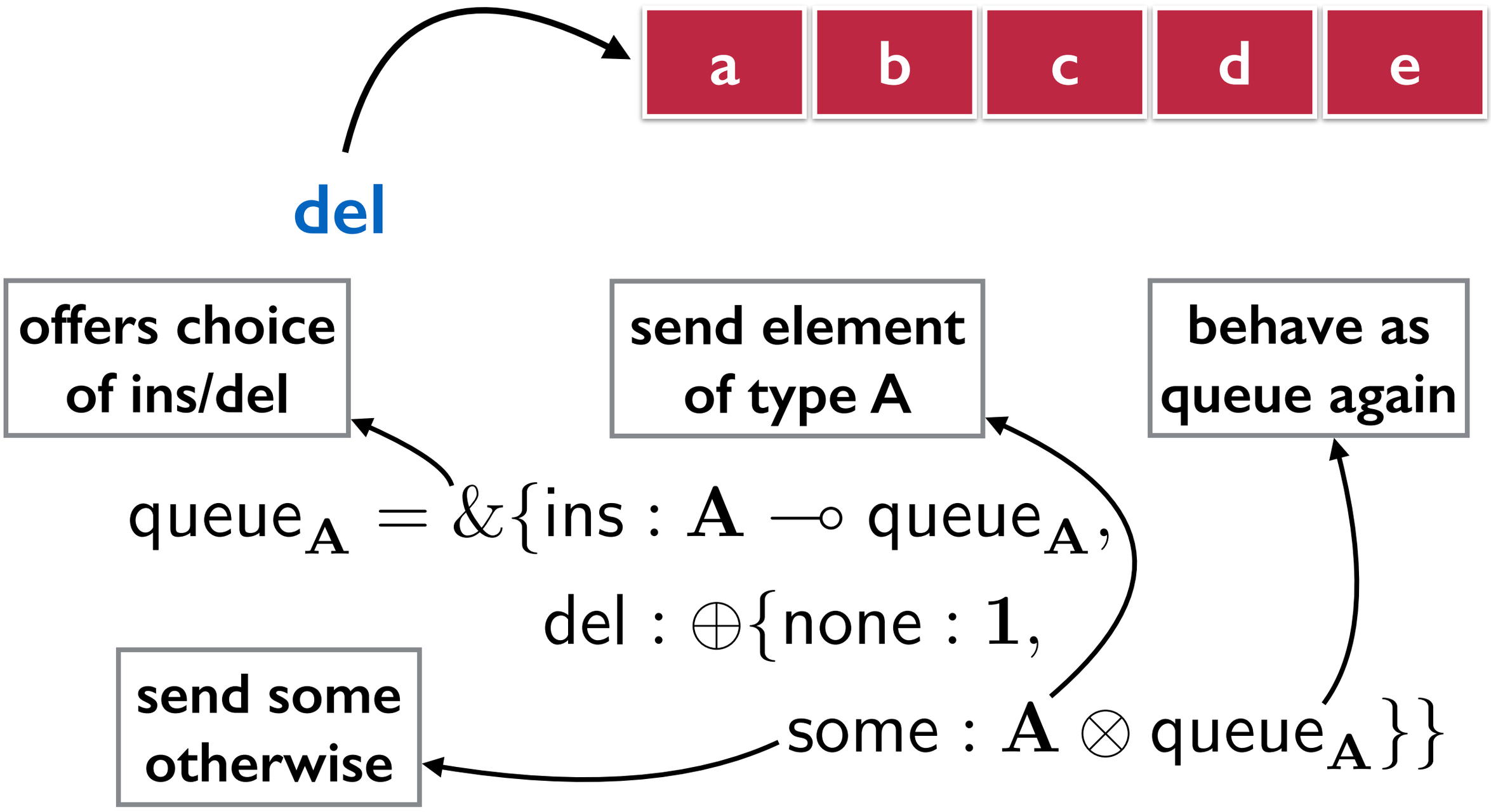

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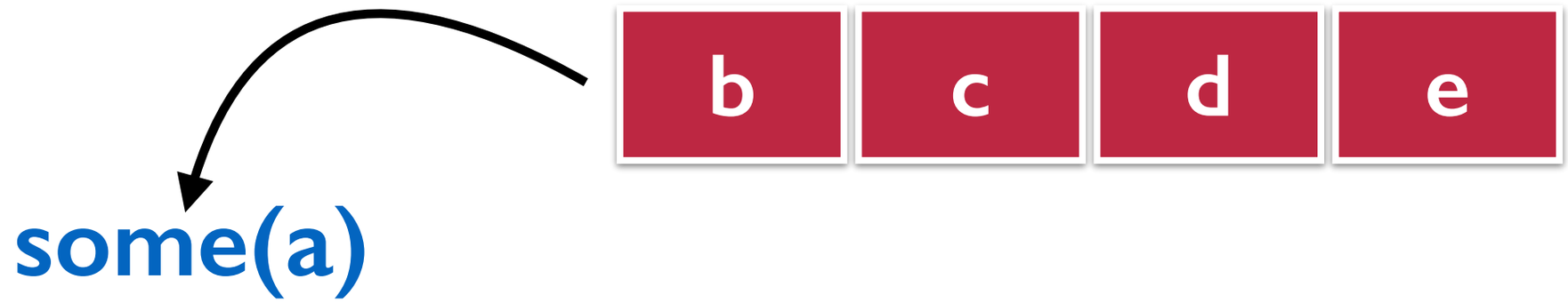

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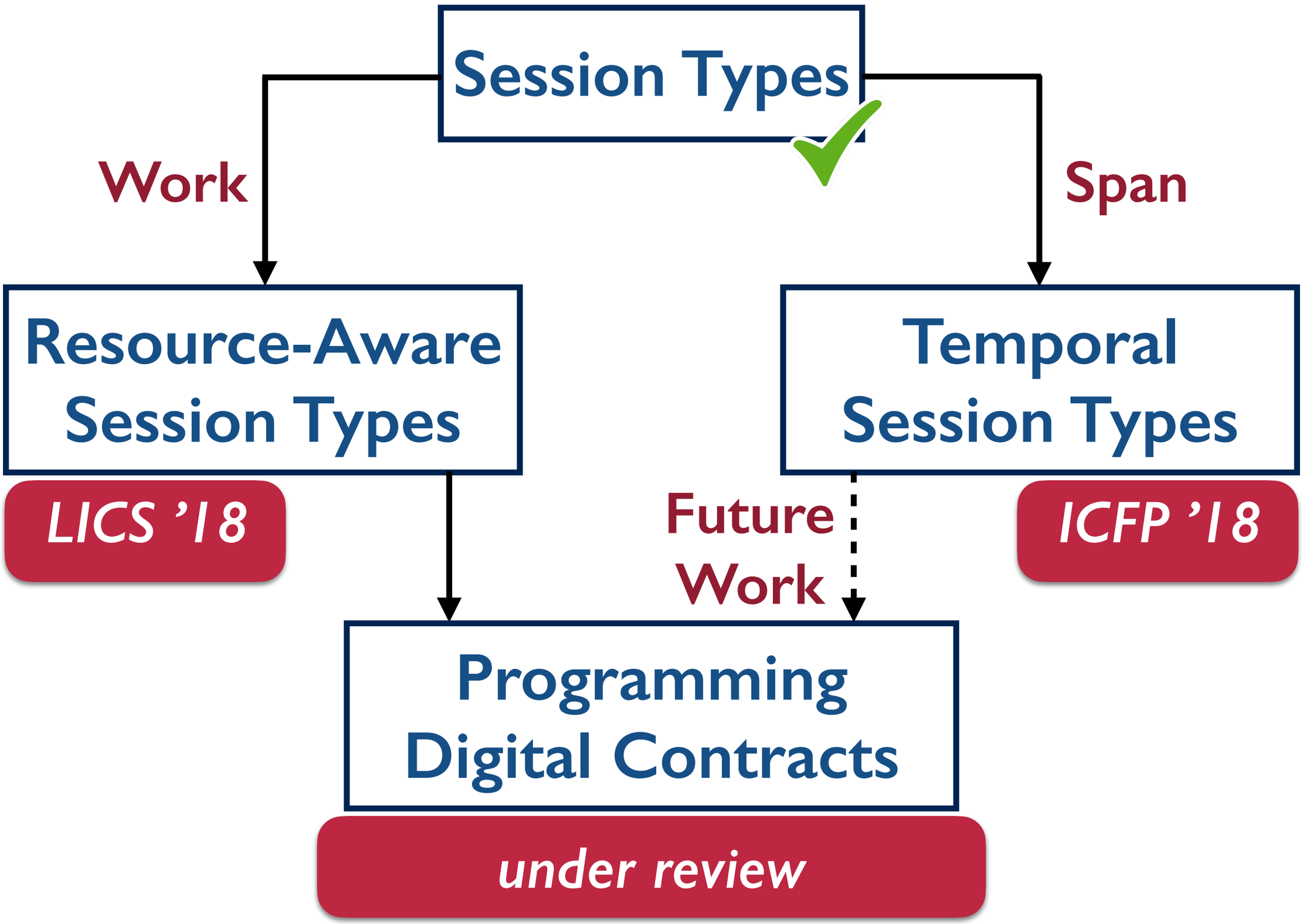

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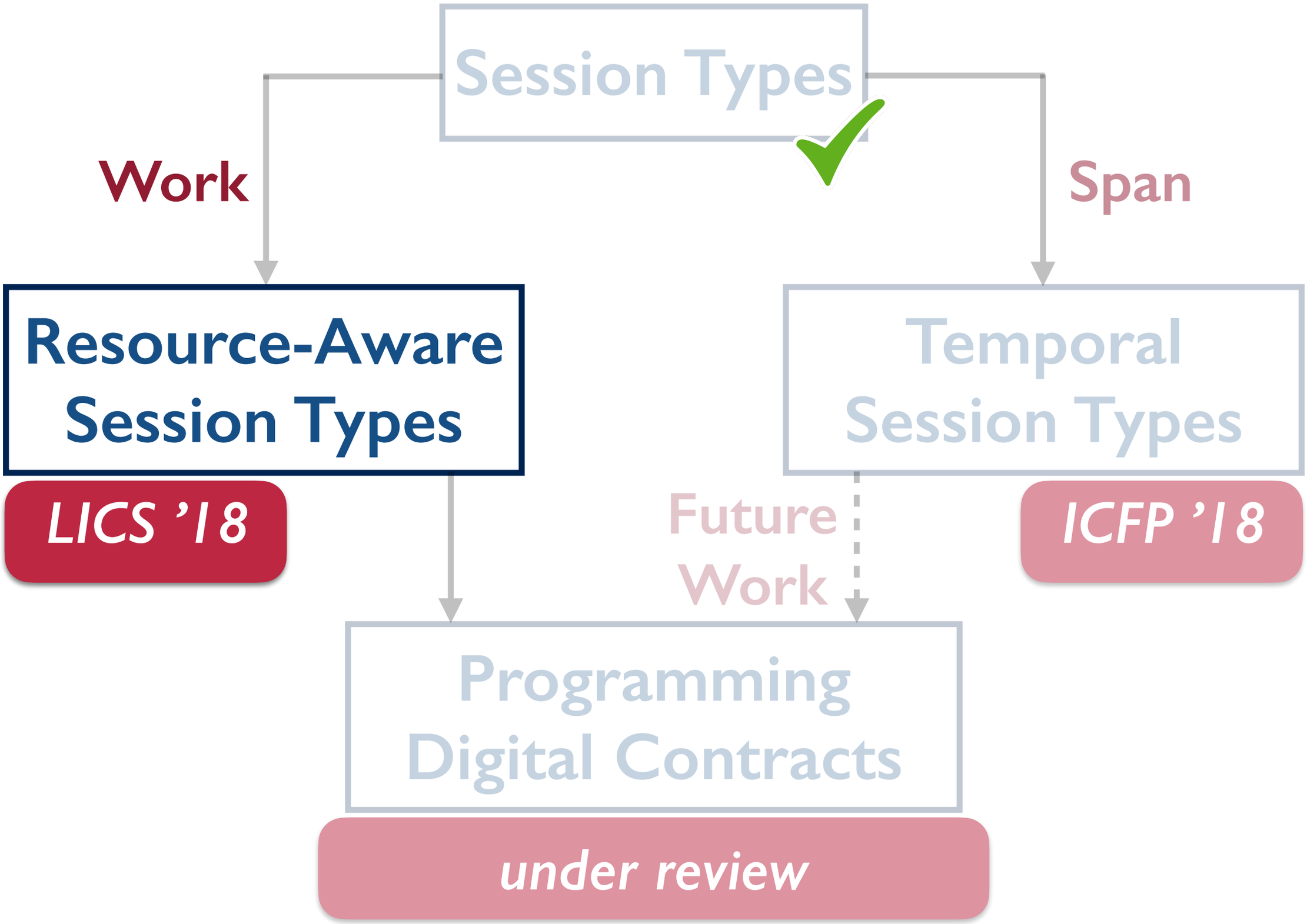

Resource Analysis

Concurrent Programs

Resource Analysis

Concurrent Programs

Work
Sequential Complexity

Execution time
on one processor

Resource Analysis

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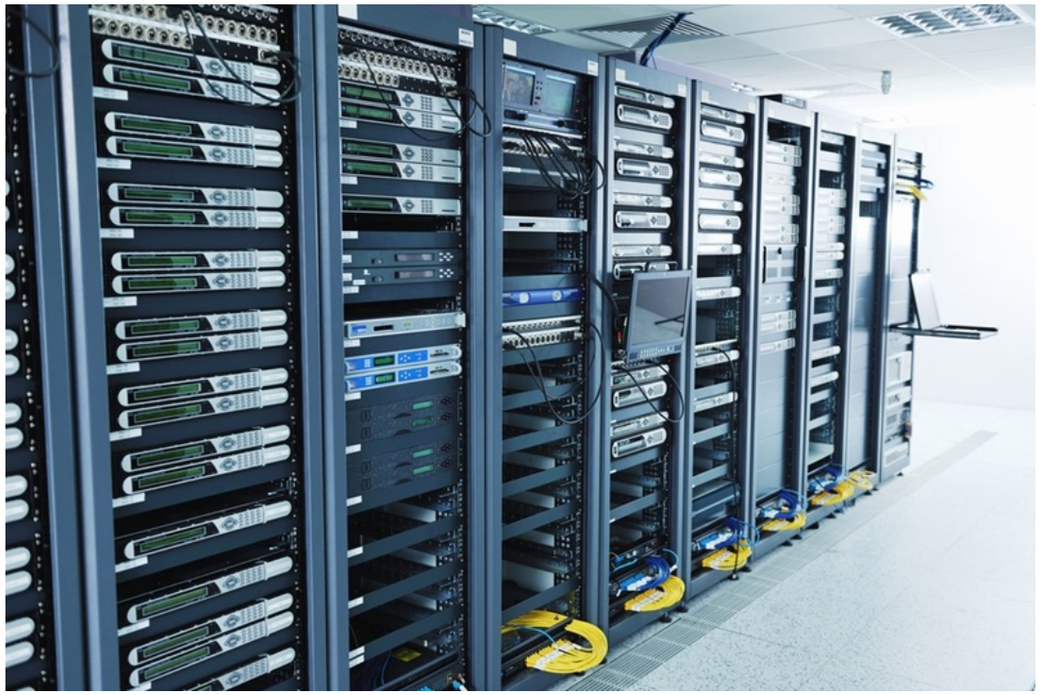

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Work done by Queue

Count the total number of messages!

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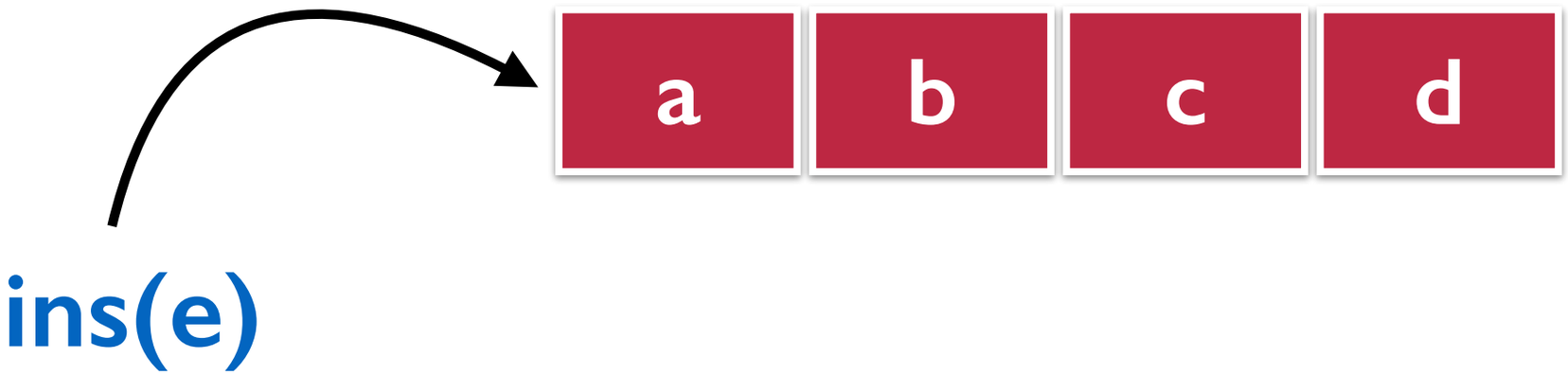

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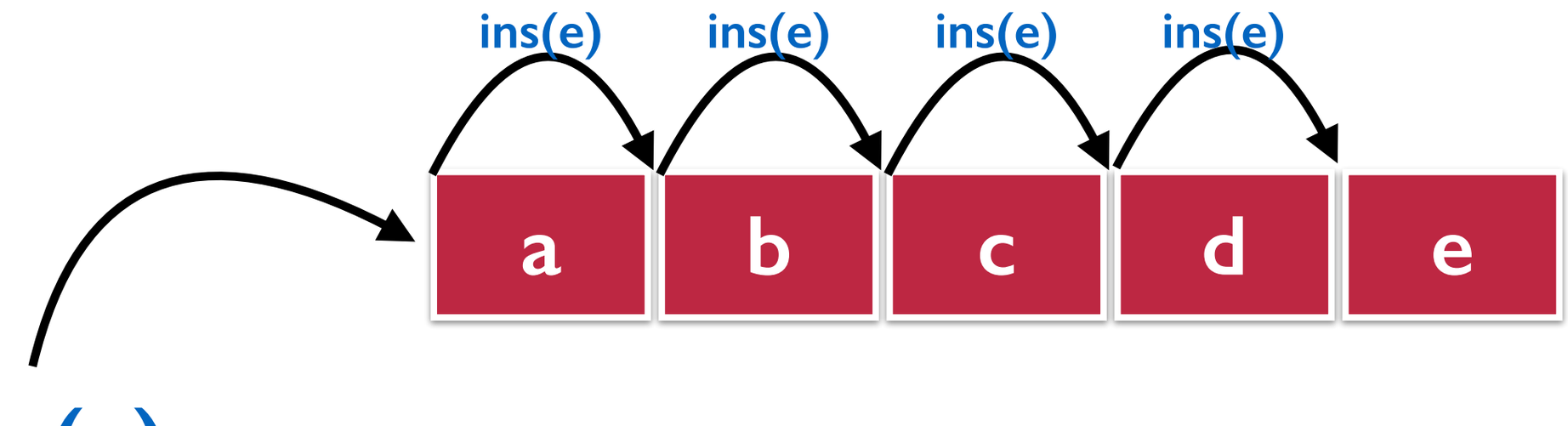

w_i = Work done to process insertion
= 2n (n is the size of queue)
= ‘ins’ and ‘e’ travel to end of queue

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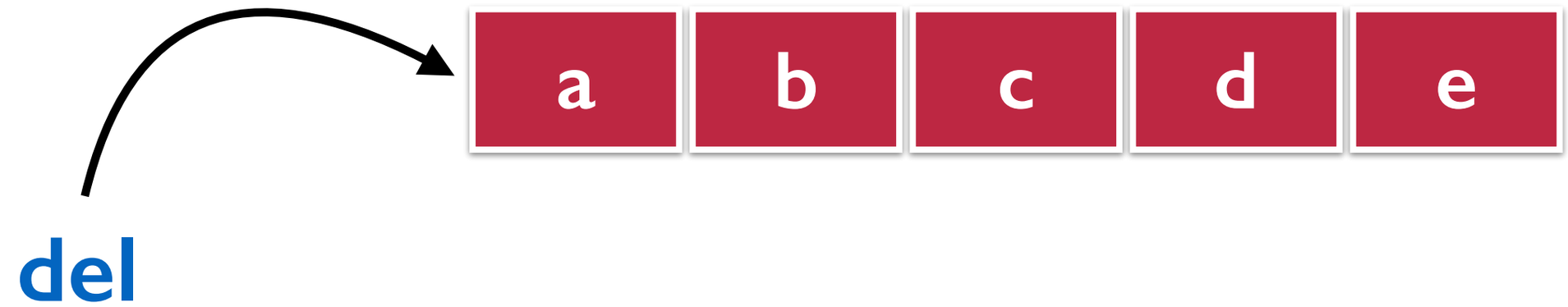

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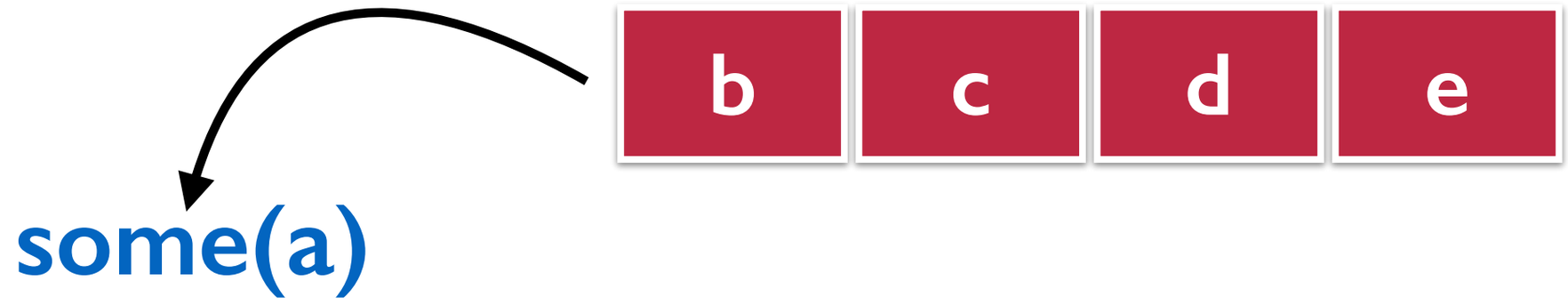

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w_d = Work done to process deletion
= 2 (sends back ‘some’ and ‘a’)

Potential Method

- Processes store potential

- Potential is exchanged via messages

- Potential is consumed to perform ‘work’

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only at type level
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User defined cost model
This talk: number of messages

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Index Refinement (Size of Queue)

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Index Refinement (Size of Queue)

Potential Annotation

- receive 2n units of potential after ‘ins’
- receive 2 units of potential after ‘del’
- potential is consumed to exchange messages

Stacks vs Queues

$$\mathbf{stack_A[n]} = \&\{\mathsf{ins} : \mathbf{A} \multimap \mathbf{stack_A[n+1]},$$
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Which one's more efficient?

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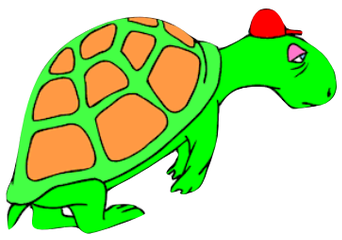

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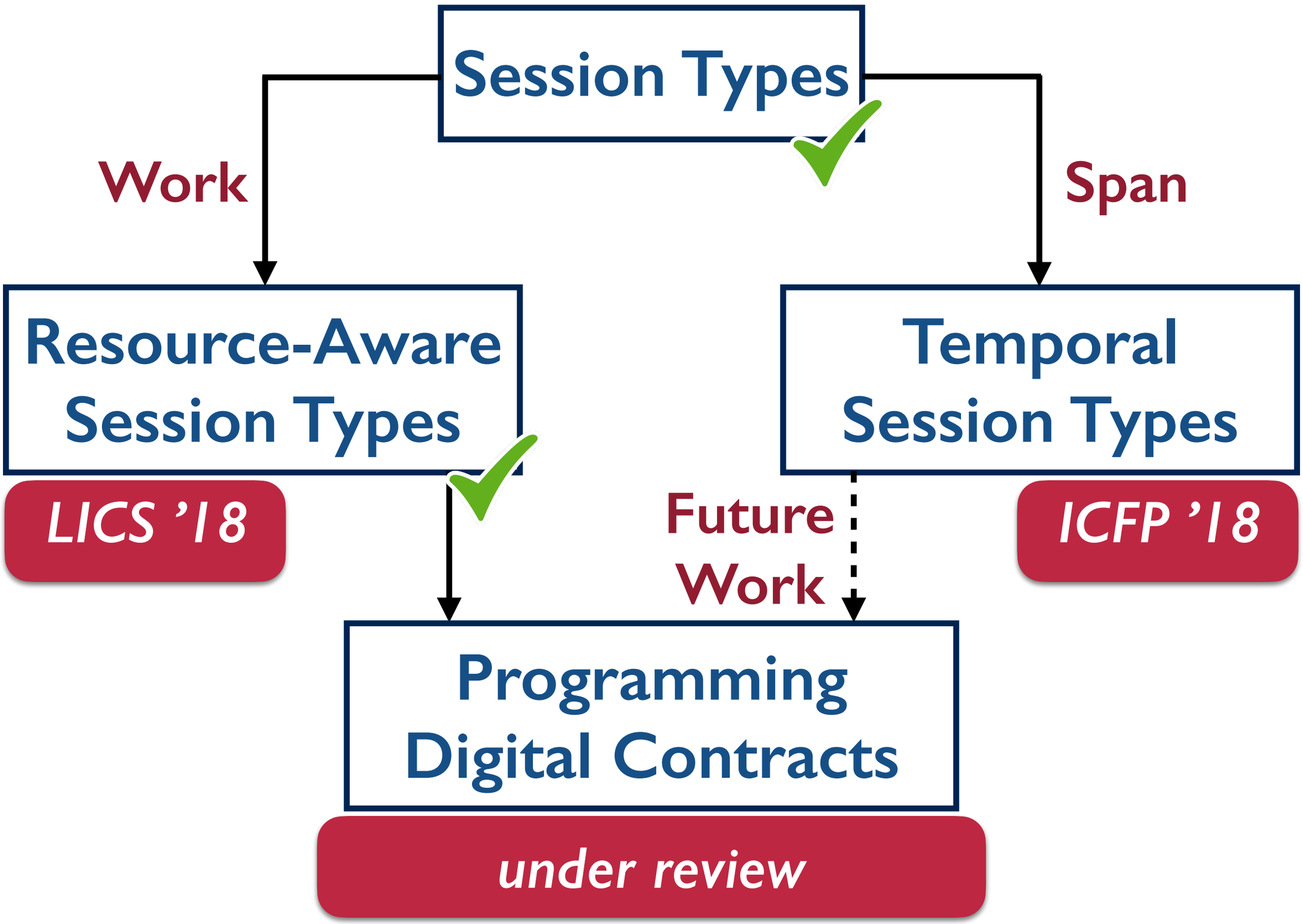

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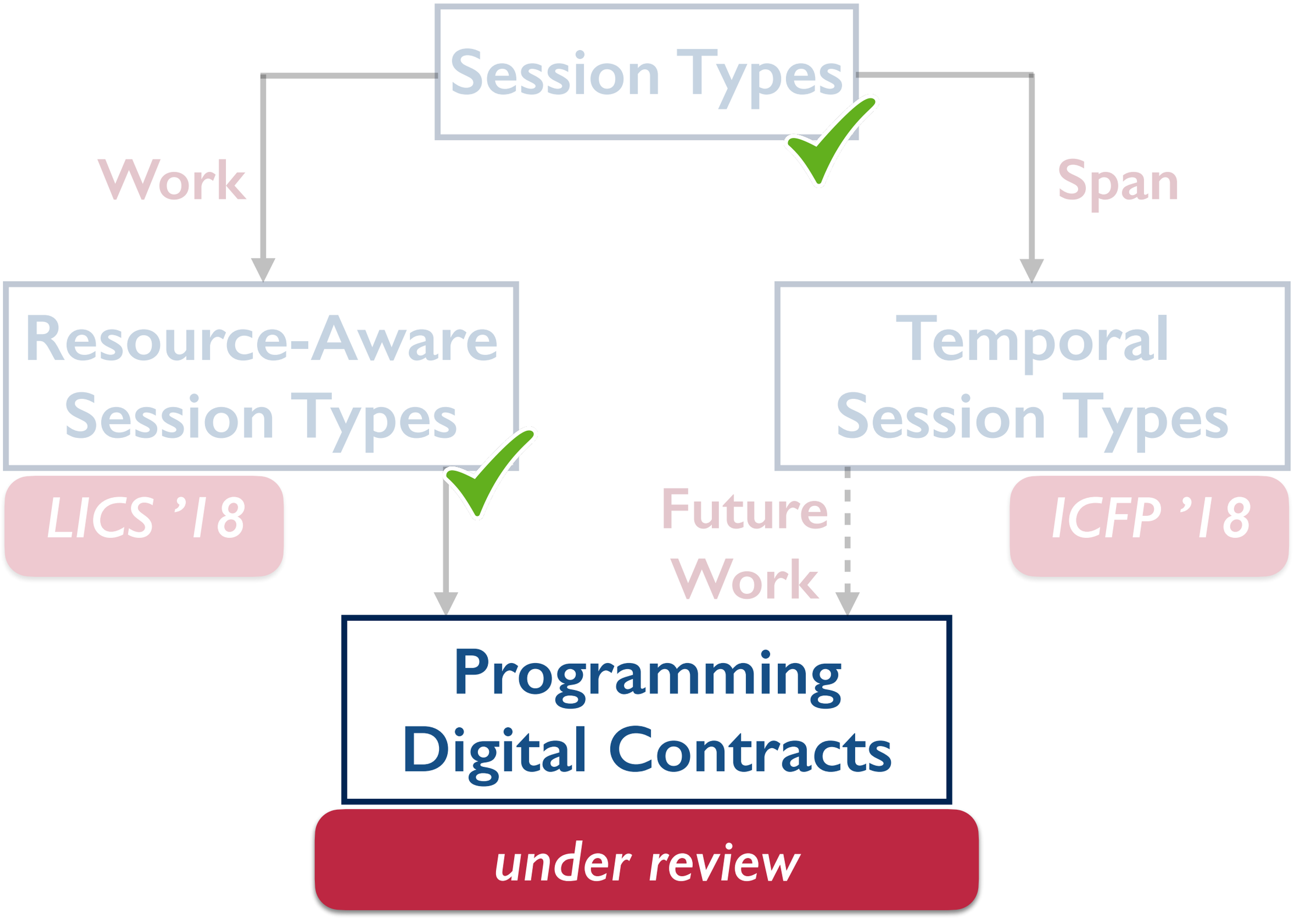

Limitations of Session Types

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Two Key Challenges

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Explored in prior work, but never combined!

Shared Channels

Balzer and Pfenning, ICFP 2017

- Types stratified into linear and shared layers
- Modal operators connecting the layers

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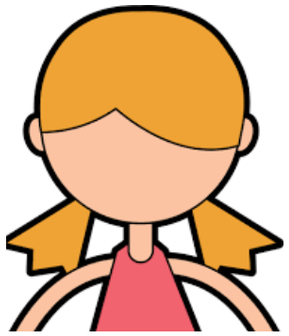
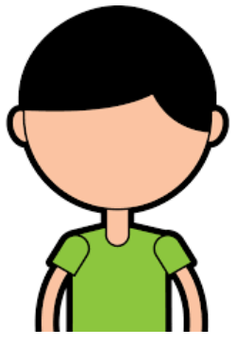
$\uparrow^{\mathsf{S}}_{\mathsf{L}} \mathbf{A_L} \Rightarrow$ **Shifts a linear type to shared**

$\downarrow^{\mathsf{S}}_{\mathsf{L}} \mathbf{A_S} \Rightarrow$ **Shifts a shared type to linear**

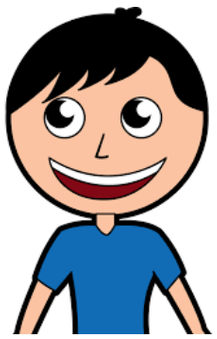
Shared Auction



Bidder 1

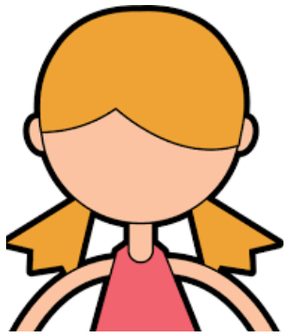


Bidder 2



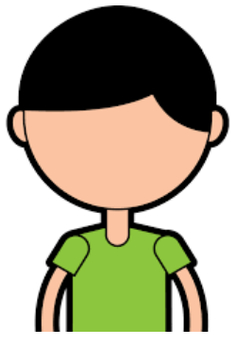
Bidder 3

Shared Auction

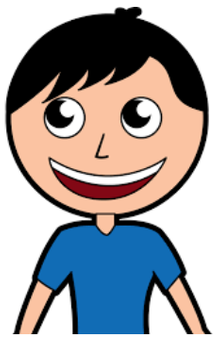


acquire!

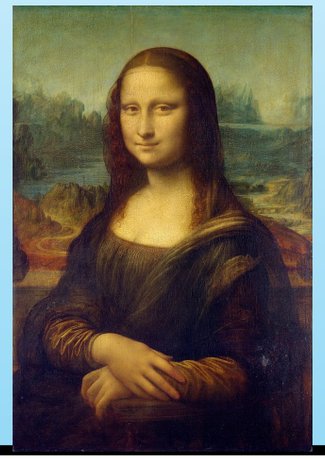
Bidder 1



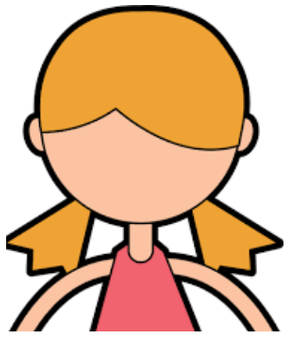
Bidder 2



Bidder 3



Shared Auction



acquire!

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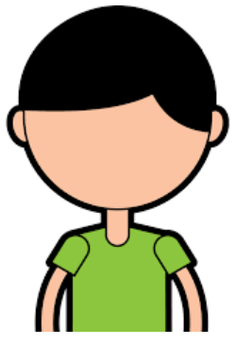
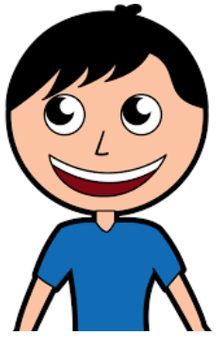


Bidder 2



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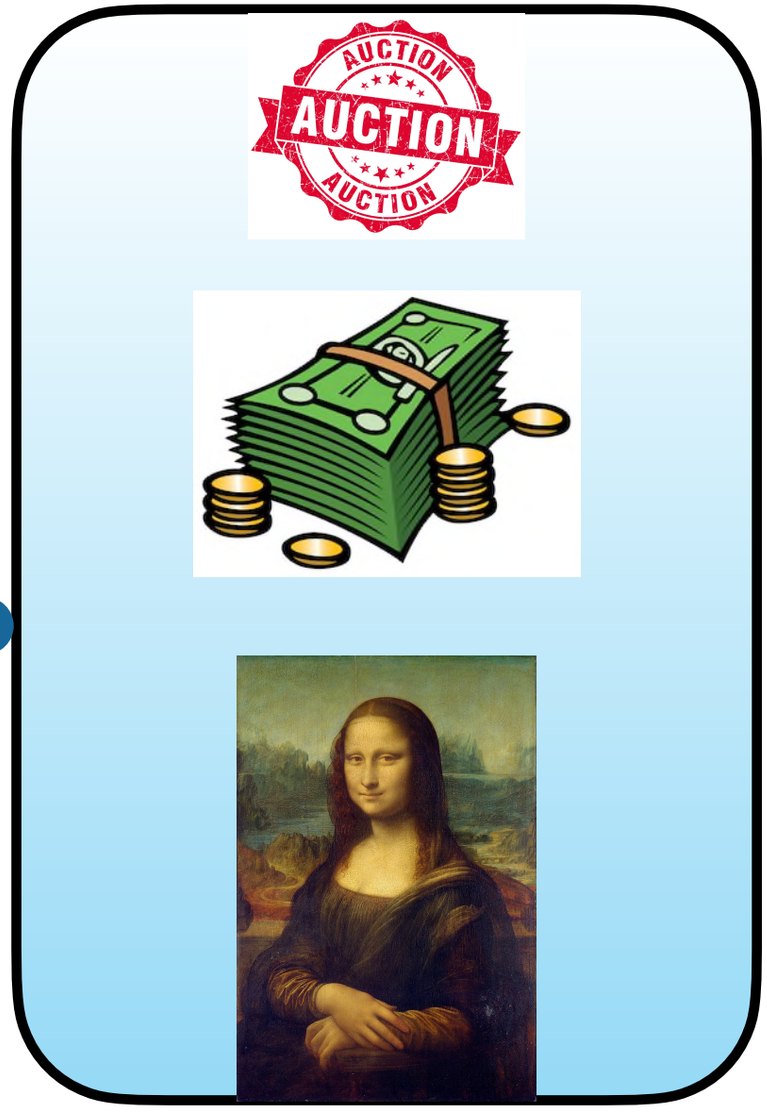

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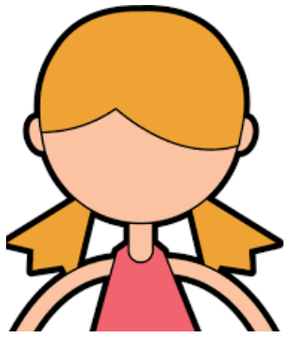
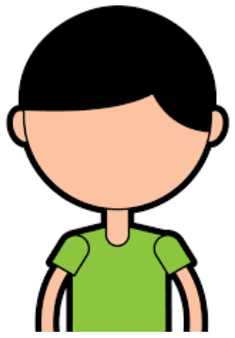
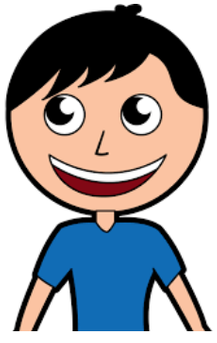


release!

Bidder 1



Bidder 2



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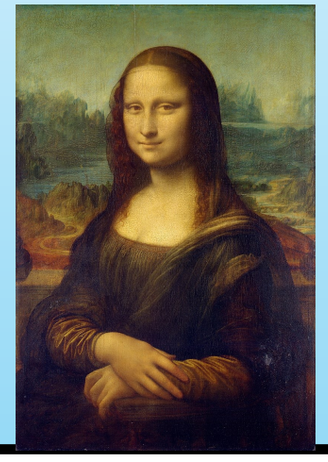

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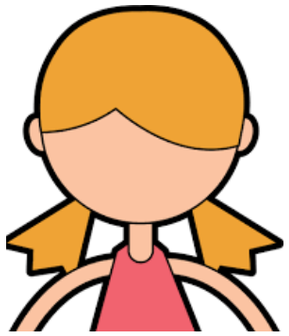

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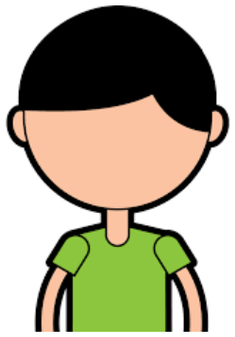
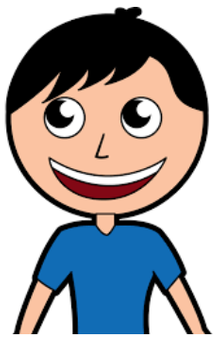
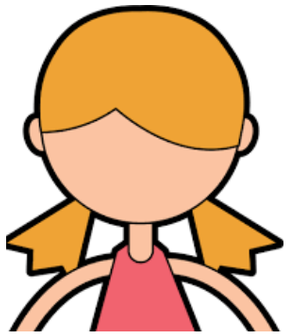


Bidder 2



Bidder 3

Shared Auction



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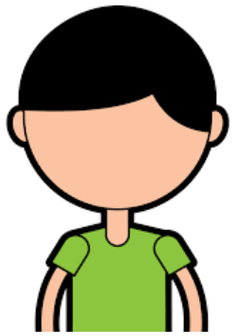

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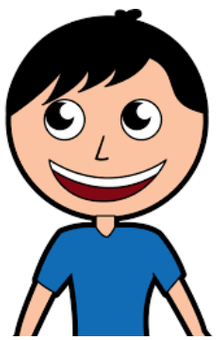

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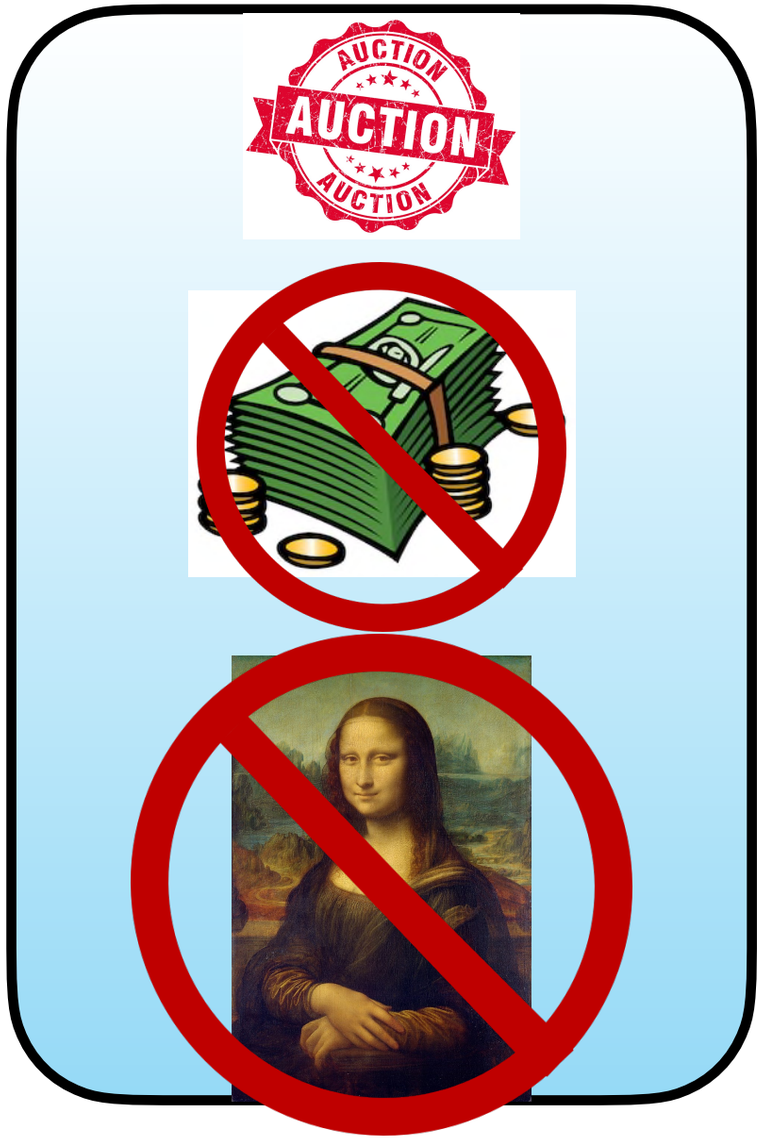

Limitation (Independence Principle): shared process cannot store any linear assets (no money in the auction contract)

Shared Auction

release!

Bidder 1

Bidder 2

Bidder 3

Prevents Re-entrancy Attacks

Limitation (Independence Principle): shared process cannot store any linear assets (no money in the auction contract)

Functional Layer

Toninho et. al., ESOP 2013

- Integrate session types in a functional programming language via a linear contextual monad
- Functional data structures isolated in a separate context in the typing judgment
- In my case: integration with Resource-Aware ML (Hoffmann, Das and Weng, POPL '17)

Shared Auction Type

$$\text{auction} = \uparrow^{S}_{L} \triangleleft^{*} \oplus \{\textbf{running} : \&\{\textbf{bid} : \text{id} \to \text{money} \multimap \triangleright^{*} \downarrow^{S}_{L} \text{auction}\},$$
$$\textbf{ended} : \&\{\textbf{collect} : \text{id} \to \oplus\{\textbf{won} : \text{monalisa} \otimes \downarrow^{S}_{L} \text{auction},$$
$$\textbf{lost} : \text{money} \otimes \triangleright^{*} \downarrow^{S}_{L} \text{auction}\}\}\}$$

*Type checker fills in * annotations automatically*

Shared Auction Type

$$\begin{aligned}\text{auction} = \uparrow^{S}_{L} \triangleleft^{22} \oplus \{&\textbf{running} : \&\{\textbf{bid} : \text{id} \to \text{money} \multimap \triangleright^{7} \downarrow^{S}_{L} \text{auction}\},\\ &\textbf{ended} : \&\{\textbf{collect} : \text{id} \to \oplus\{\textbf{won} : \text{monalisa} \otimes \downarrow^{S}_{L} \text{auction},\\ &\qquad\qquad \textbf{lost} : \text{money} \otimes \triangleright^{2} \downarrow^{S}_{L} \text{auction}\}\}\}\end{aligned}$$

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Shared Auction Type

shared: contract is acquired before use

$$\text{auction} = \uparrow^{S}_{L} \triangleleft^{22} \oplus \{\textbf{running} : \&\{\textbf{bid} : \text{id} \to \text{money} \multimap \triangleright^{7} \downarrow^{S}_{L} \text{auction}\},$$
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receive 22 units of potential

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receive 22 units of potential

send back 7 units of potential

*Type checker fills in * annotations automatically*

Shared Auction Type

shared: contract is acquired before use

shared: contract is released after use

$$\text{auction} = \uparrow^{S}_{L} \triangleleft^{22} \oplus \{\textbf{running} : \&\{\textbf{bid} : \text{id} \to \text{money} \multimap \triangleright^{7} \downarrow^{S}_{L} \text{auction}\},$$
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Running Auction

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$sa \leftarrow \textit{run}\ b \leftarrow M\ l =$

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$la.\text{running}\ ;$

$\text{case}\ la$

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$m.\text{value}\ ;$

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$b' = \textit{addbid}\ b\ (r, v)\ ;$

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accept ‘acquire’ ($\uparrow^{S}_{L}$)

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accept ‘acquire’ ($\uparrow^{S}_{L}$)

send status ‘running’

recv ‘id’ and ‘money’

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accept ‘acquire’ ($\uparrow^{S}_{L}$)

send status ‘running’

recv ‘id’ and ‘money’

detach from client ($\downarrow^{S}_{L}$)

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recv ‘id’ and ‘money’

detach from client ($\downarrow^{S}_{L}$)

add bid and money

no work constructs!

Programming Contracts

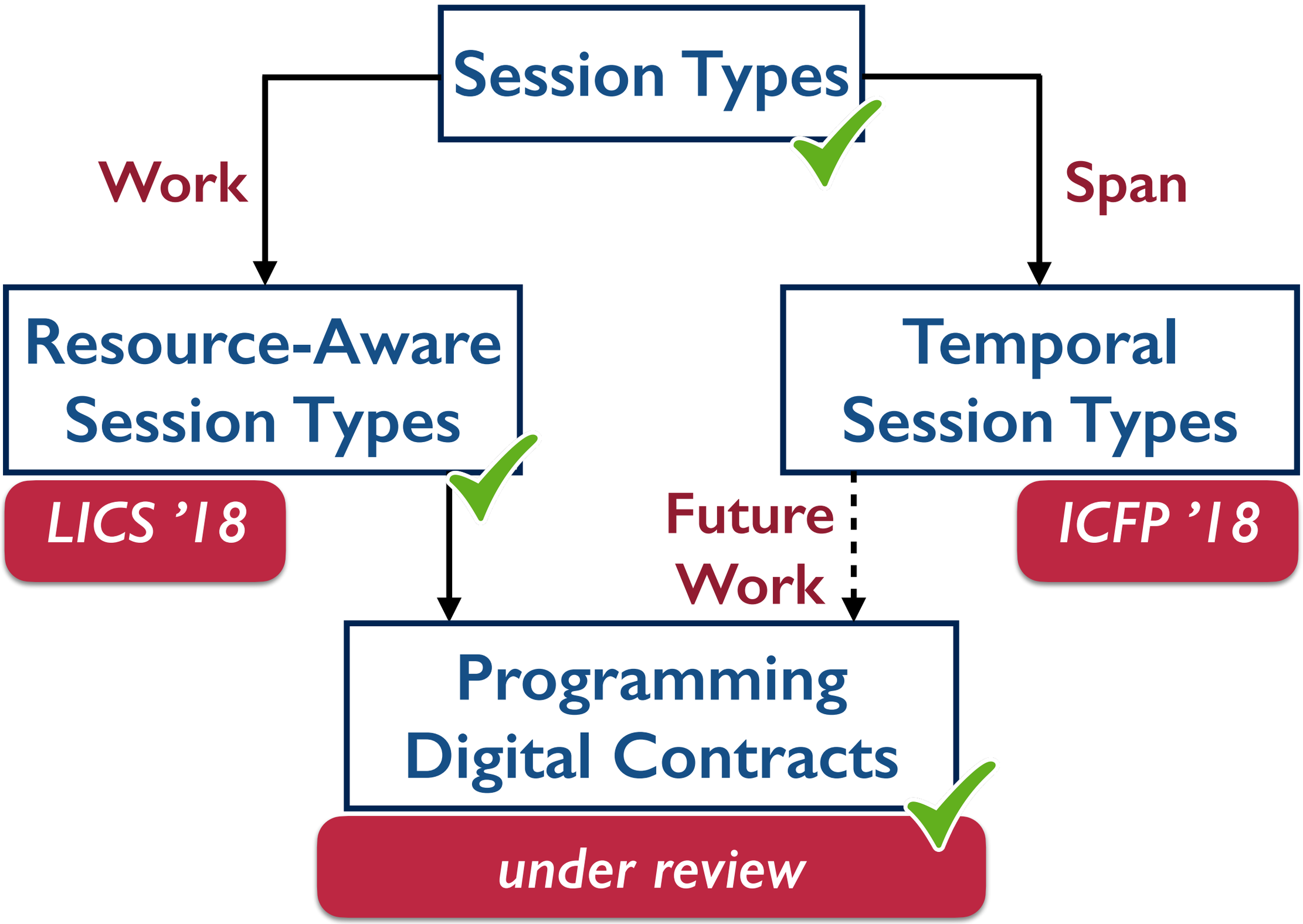
Existing Languages (e.g. Solidity)

- Protocol not explicit in code, enforced programmatically
- Resource (aka gas) usage not analyzed
- Linearity of assets (money) not enforced
- Prone to re-entrancy

Proposed Language (Nomos)

- Session types express protocol, enforced by type checking
- Resource-aware types express gas usage
- Linear type system tracks assets
- No re-entrancy attack

Talk Outline



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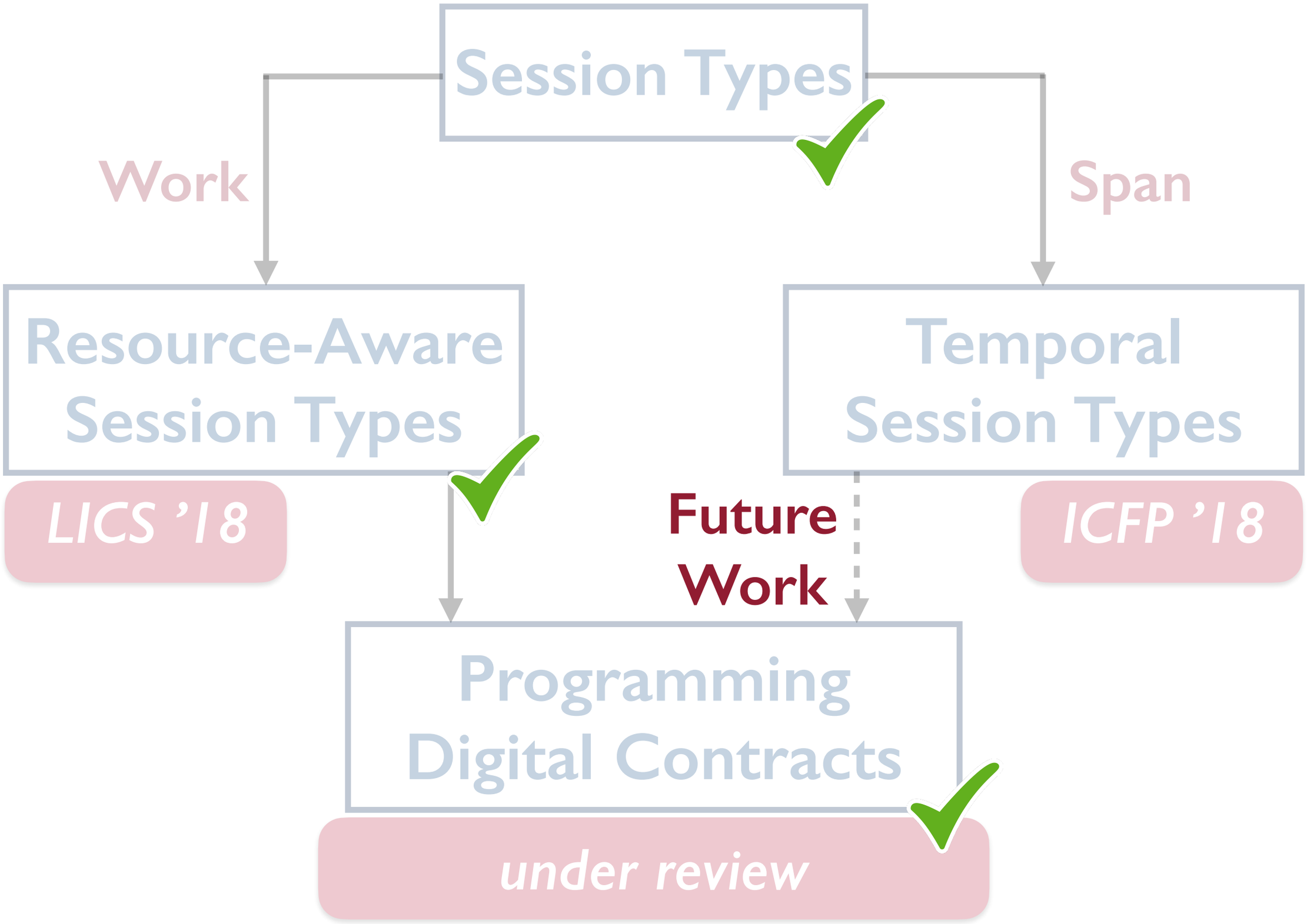

Future Directions

- *Tracking time* in Nomos for time-specific contracts
- Evaluation of *efficiency* and *scalability* of Nomos
- *Runtime monitoring* to ensure Nomos contracts can interact with ill-typed and untyped clients
- Deadlock detection of session-typed programs
- Integrating *refinement types* to prove stronger invariants (e.g. money bid is equal to money returned) (under review)

Conclusion

- **Resource-Aware Session Types:** track sequential complexity using potential method
- **Temporal Session Types:** track parallel complexity using temporal operators

- Resource-aware session types are great for implementing digital contracts
- Types express contract protocol, track resource usage, enforce linearity of assets, prevent re-entrancy

Typing Judgment

$$\Psi \; ; \; \Gamma \; ; \; \Delta \; \vdash^{q} P :: (x : A)$$

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Functional Context

- All structural rules
- Copying semantics
- copied during exchange

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Typing Judgment

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Functional Context

- All structural rules
- Copying semantics
- copied during exchange

Shared Context

- All structural rules
- Shared Semantics
- no copying of channels

Linear Context

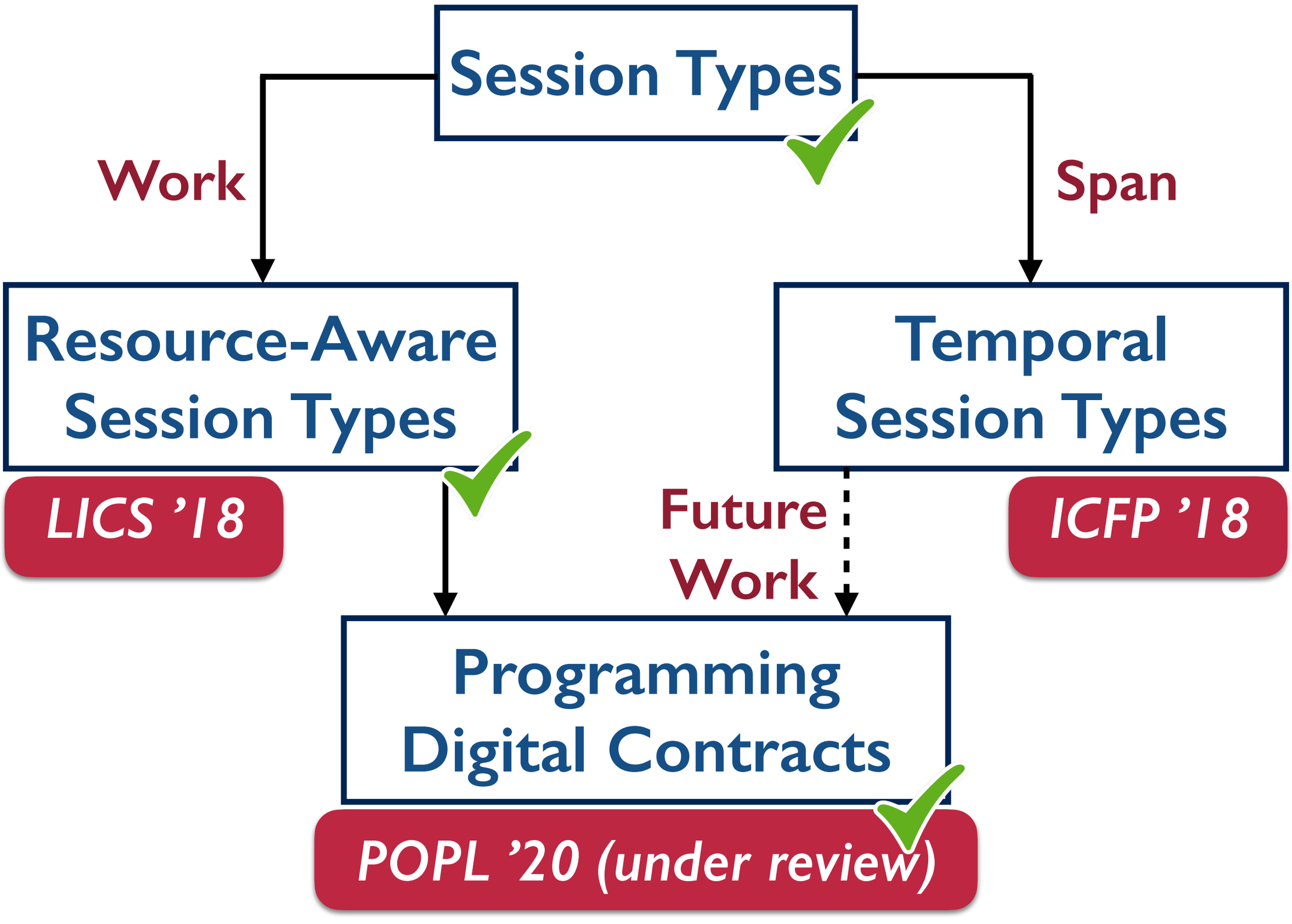
- Only exhibits exchange (no weakening or contraction)
- can't discard or duplicate

Relaxing Independence

- Distinguish linear processes according to their roles
  - Assets : can only refer to other linear assets ⇒ assign mode R (e.g. money, Mona Lisa)
  - Contracts : can refer to other contracts or linear assets ⇒ assign mode L (e.g. auction)
  - Transactions : can refer to assets, contracts and transactions ⇒ assign mode T (e.g. bidder)

$$R < L < T$$

Talk Outline



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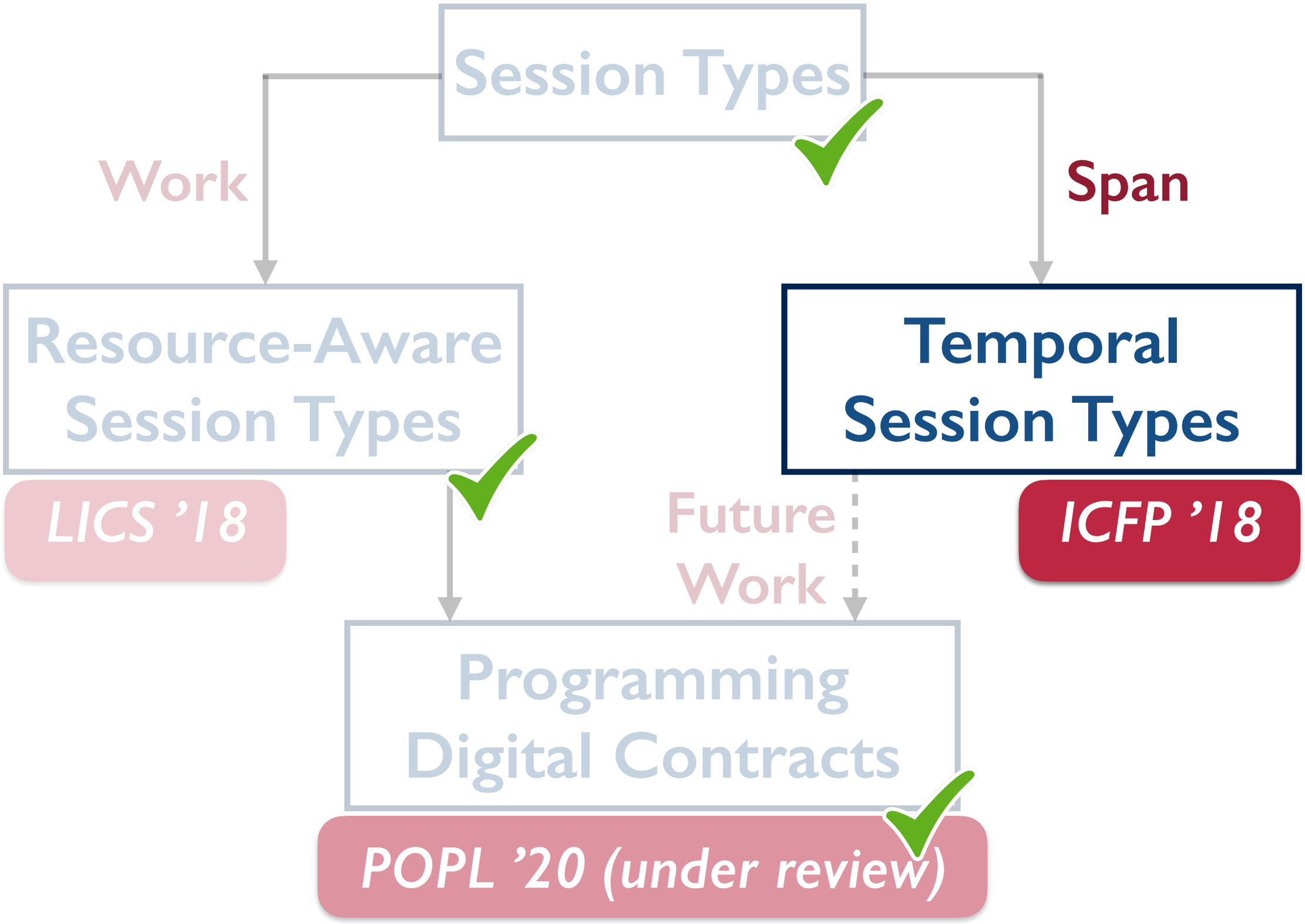

How is time defined?

- Time is defined using a cost model
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$\mathcal{R}$ cost model	$\mathcal{RS}$ cost model
Unit delay after each receive	Unit delay after each receive and send

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$\mathcal{R}$ cost model	$\mathcal{RS}$ cost model
Unit delay after each receive	Unit delay after each receive and send

- Expressed by inserting appropriate delays in the source code, only the delays cost time
- Programmer specifies cost model, compiler automatically inserts delays for type checking

Example: Bit Streams

$$\mathbf{bits} = \oplus\{\mathbf{b0} : \mathbf{bits}, \mathbf{b1} : \mathbf{bits}, \$: \mathbf{1}\}$$

$$\cdot \vdash \mathbf{two} :: (\mathbf{c} : \mathbf{bits})$$

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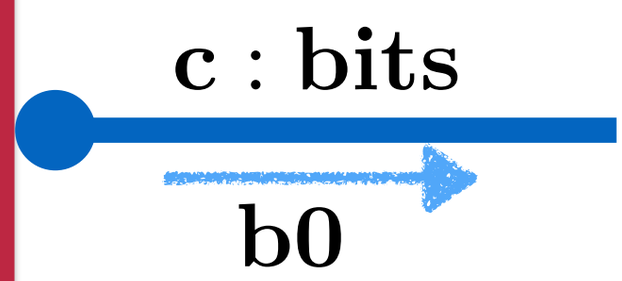

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b0

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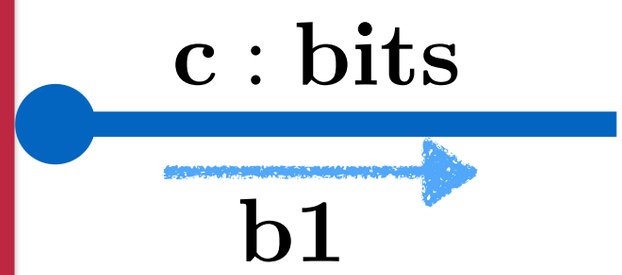

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Example: Bit Streams

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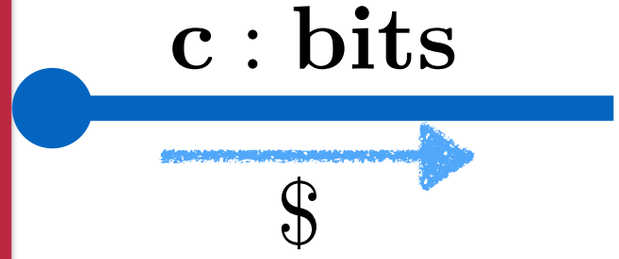

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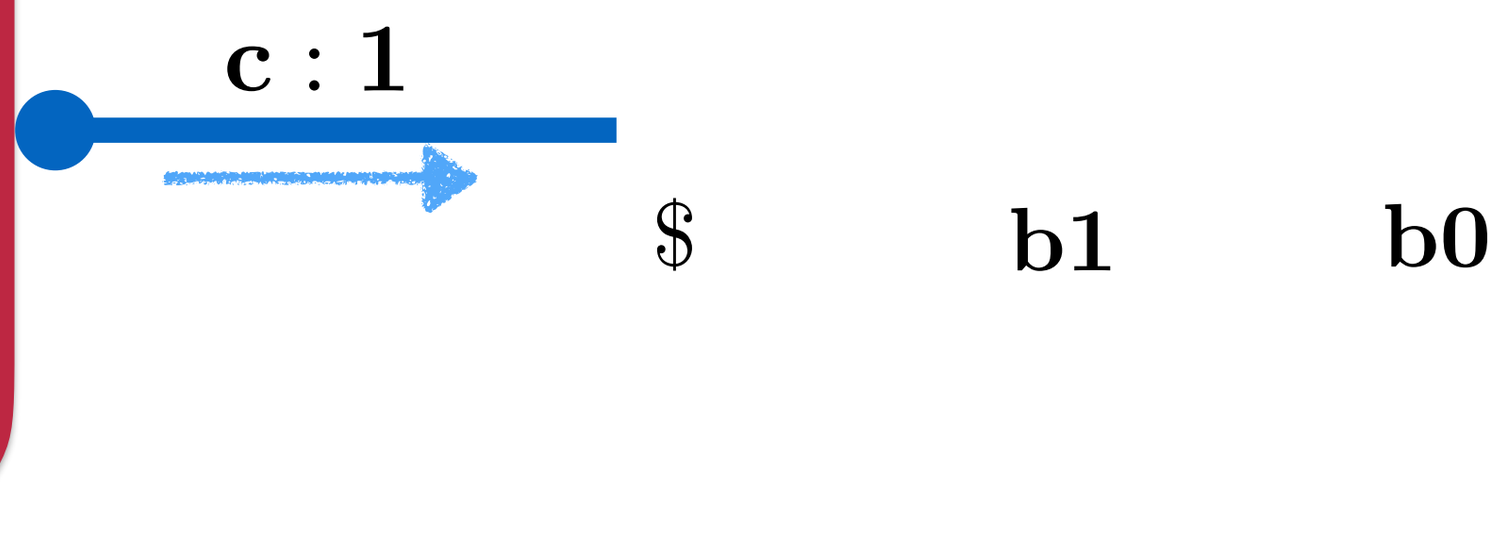

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Sending a message causes unit delay

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$\mathbf{close}$ $\quad \$ \quad \mathbf{b1} \quad \mathbf{b0}$

$\mathbf{t = 3} \quad \mathbf{t = 2} \quad \mathbf{t = 1} \quad \mathbf{t = 0}$

Enforcing Time in the Type

$$\mathbf{bits} = \oplus\{\mathbf{b0} : \bigcirc\mathbf{bits}, \mathbf{b1} : \bigcirc\mathbf{bits}, \$: \bigcirc\mathbf{1}\}$$

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$\mathbf{c} : \mathbf{bits}$

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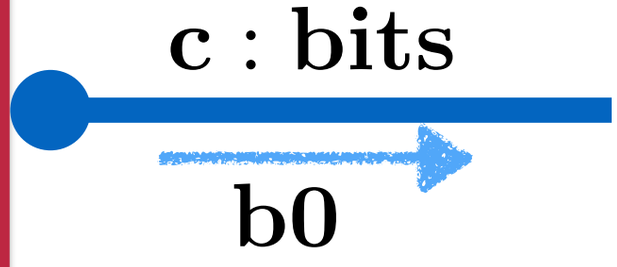

$$\mathbf{t} = \mathbf{0}$$

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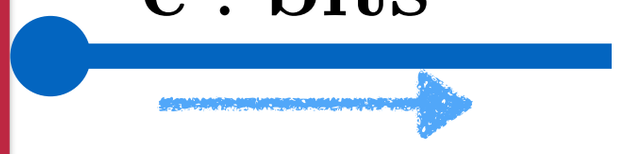

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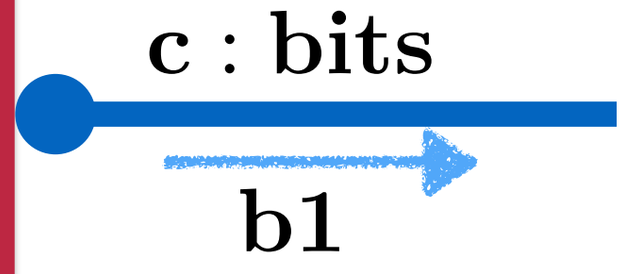

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Can we type the queue?

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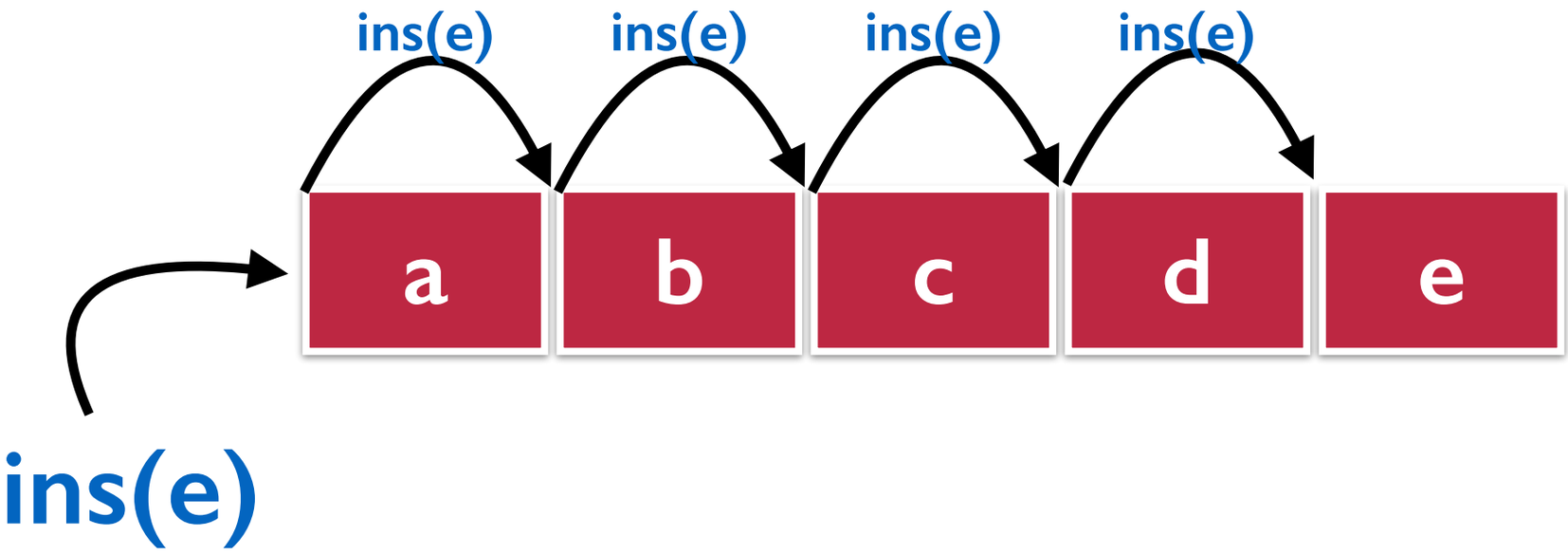

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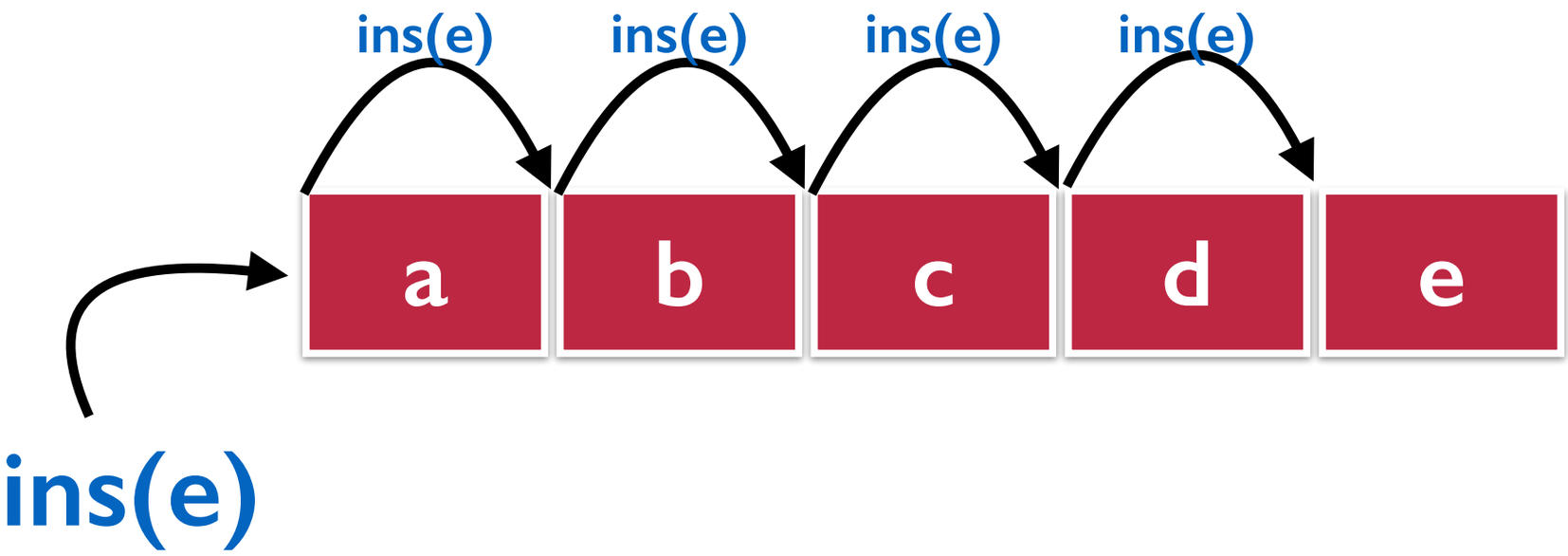

- Next operator only expresses constant insertion rate
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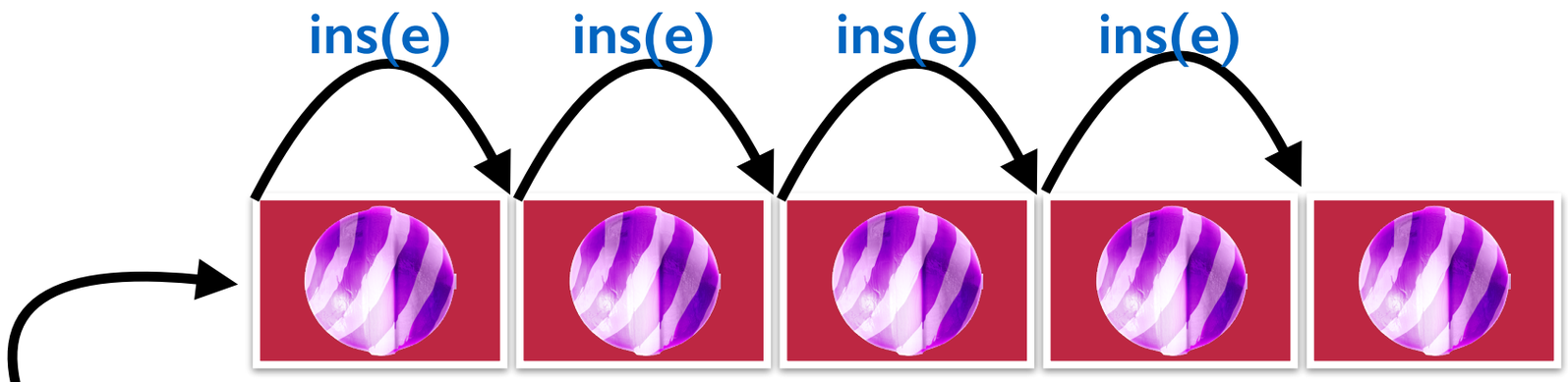

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Providing Flexibility

- **The Box Operator (□)**
  - Provider Action: always be ready to receive token
  - Client Action: eventually send the token
    - Provider doesn't know when the token will come, only the client does
    - Different from ○ operator where both provider and client knew the timing of message exchange
- **The Diamond Operator (◇)**
  - Dual of the Box operator (provider and client flip)

Response Time of Queues

$$\mathbf{queue_A} = \Box\,\&\{\mathbf{ins} : \bigcirc(\Box\mathbf{A} \multimap \bigcirc^{\mathbf{3}}\mathbf{queue_A})\},$$
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Precision

WE ARE HERE!

Flexibility

Stacks vs Queues

$\mathcal{RS}$ cost model

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Which one's more efficient?

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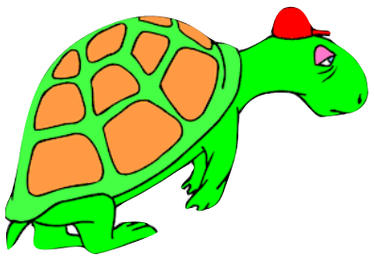

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Contributions

Type system to analyze timing of message exchanges of session-typed programs

- types define the *timing* of message exchanges
- provides *precision* and *flexibility*
- proved *sound* w.r.t. *cost semantics* tracking time
- *conservative* extension to typical session type system
- applies to all *standard* session types examples
- can be *parameterized* to count resource of interest