Work Analysis with Resource-Aware Session Types

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Jan Hoffmann
Frank Pfenning

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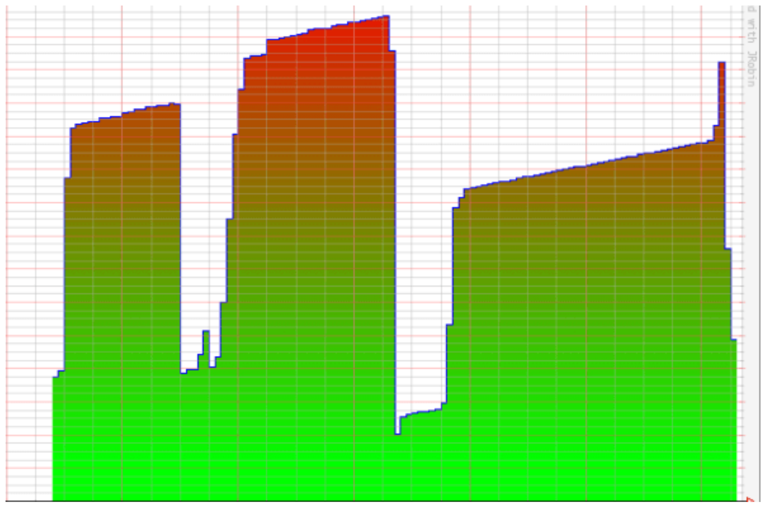
Carnegie Mellon University
Computer Science Department

Goal for this Talk

Resource Analysis
for Concurrent Programs

Execution Time



Memory Usage

Why Resource Analysis?

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Complexity of Parallel Algorithms

Çiçek et. al. (ESOP '15)

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Static and Dynamic Profiling

Haemmerlé et. al. (FLOPS ’17)

Measures of Execution Time

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Work
Sequential Complexity

Execution time on one processor

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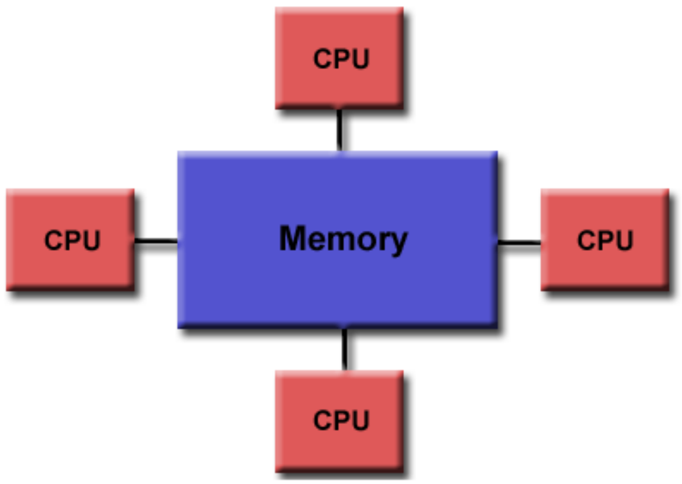

Shared Memory Read/Write Overhead

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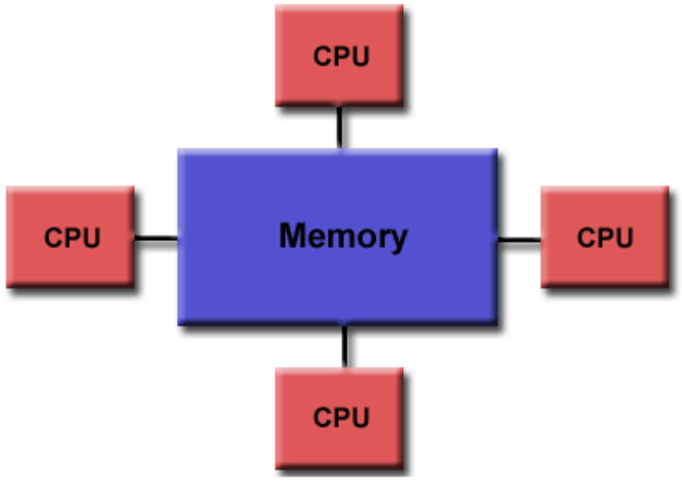

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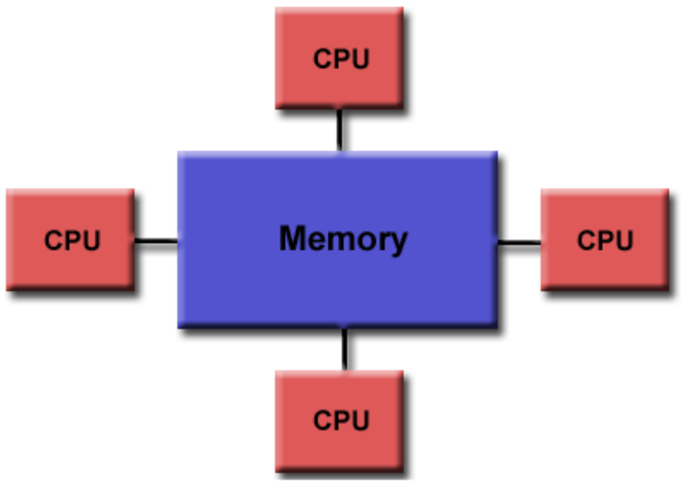

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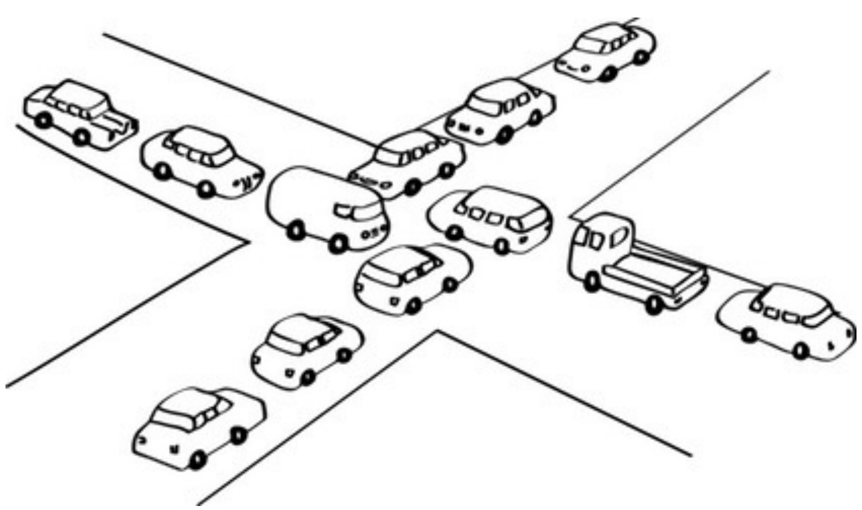

Deadlocks

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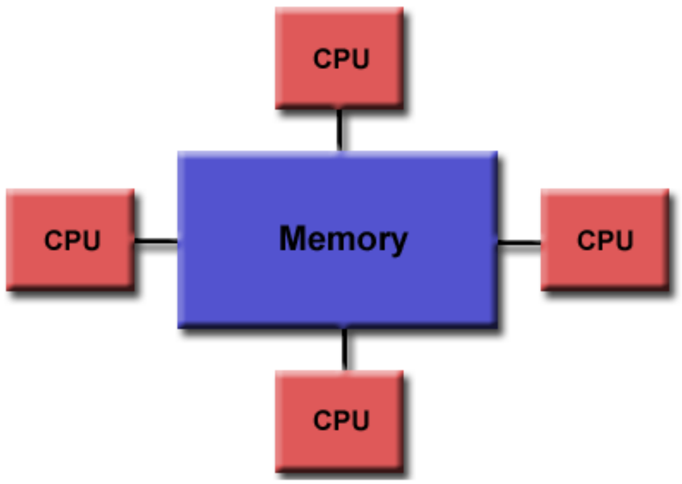

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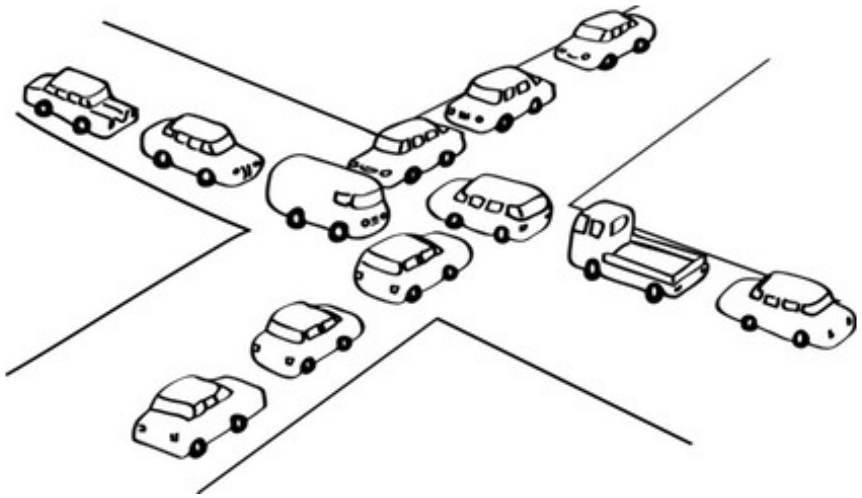
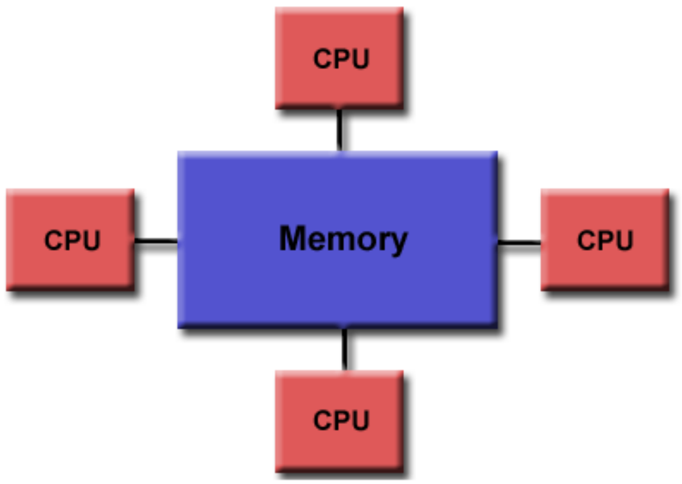


Deadlocks

Non-Compositional

Session types can help :-)



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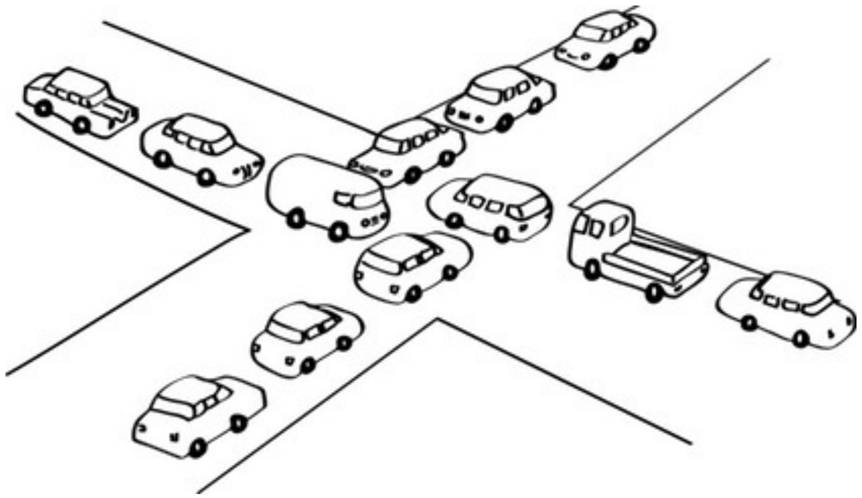

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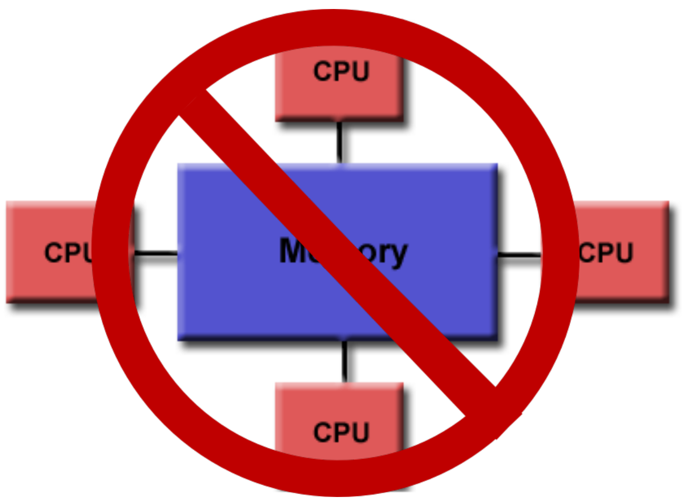

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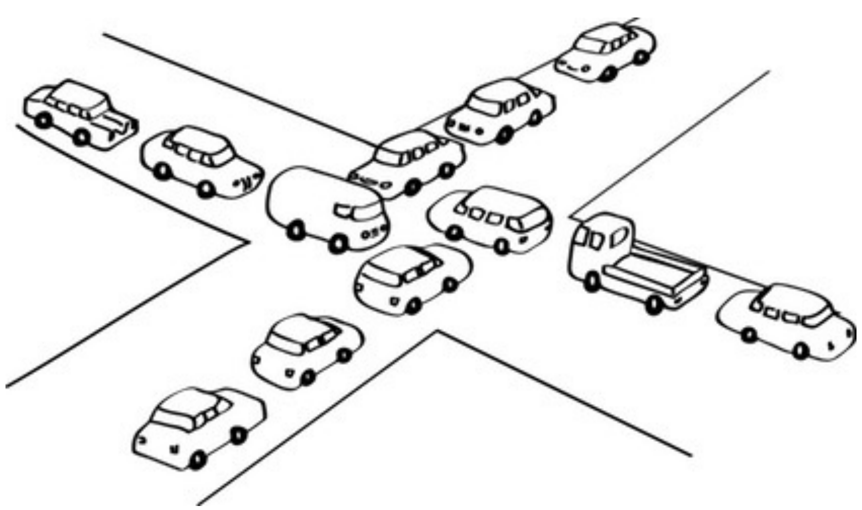

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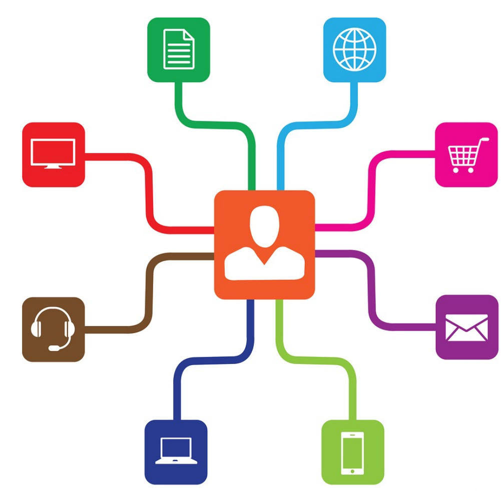

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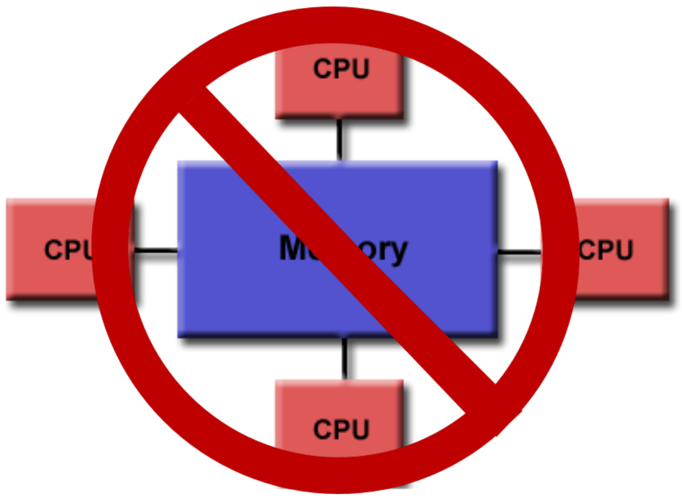

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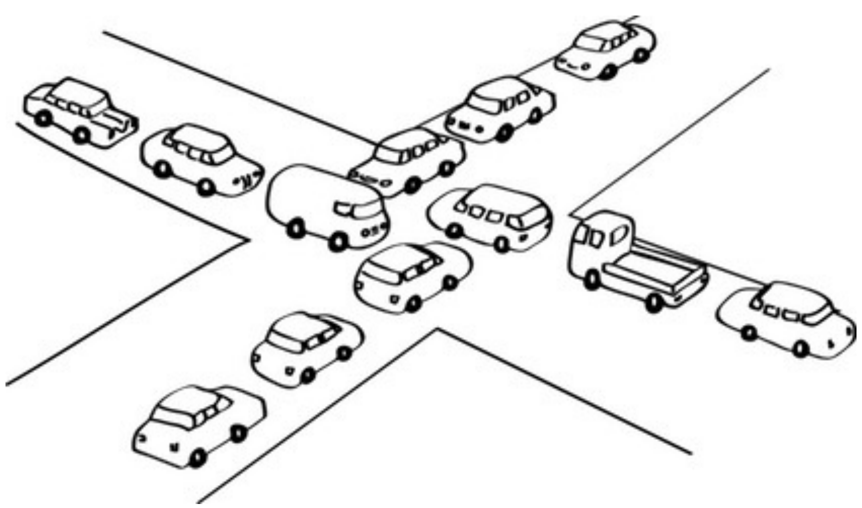
Types strictly enforce communication protocols



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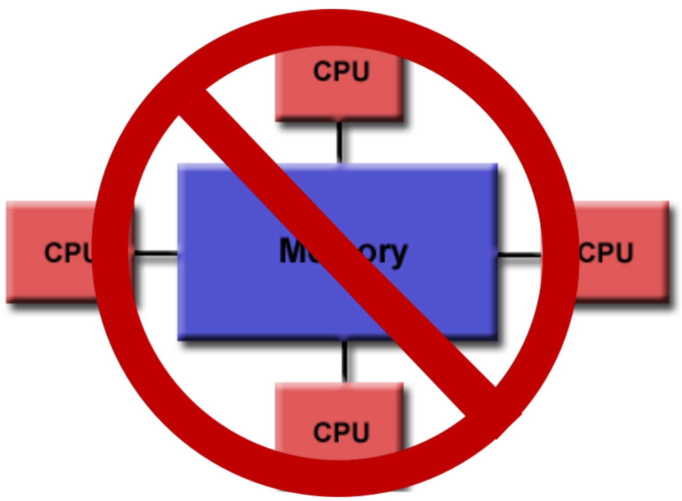

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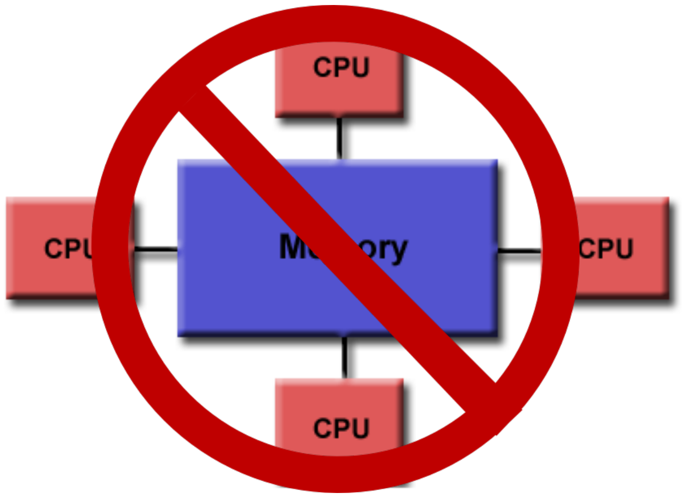

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Channels abstract over connected processes

What are Session Types?

- Implement message-passing concurrent programs
- Communication via bi-directional typed channels
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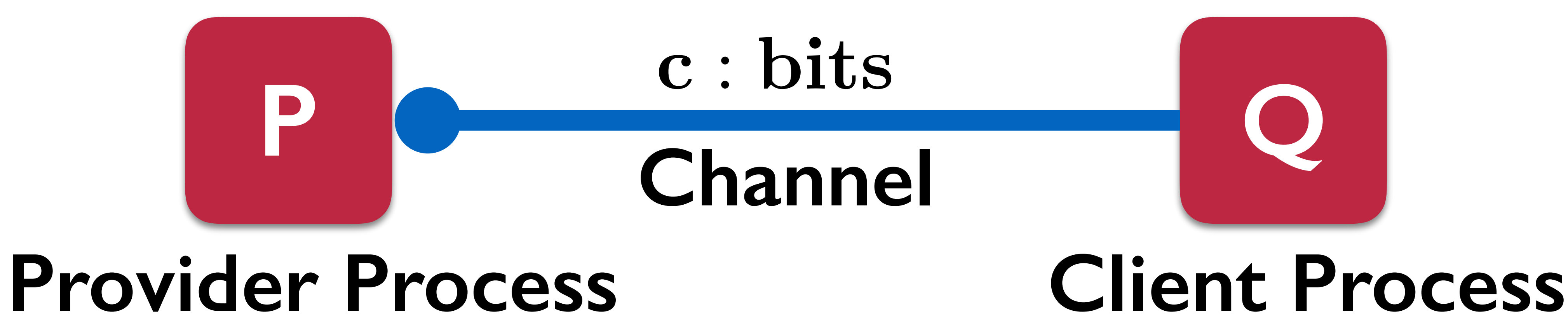

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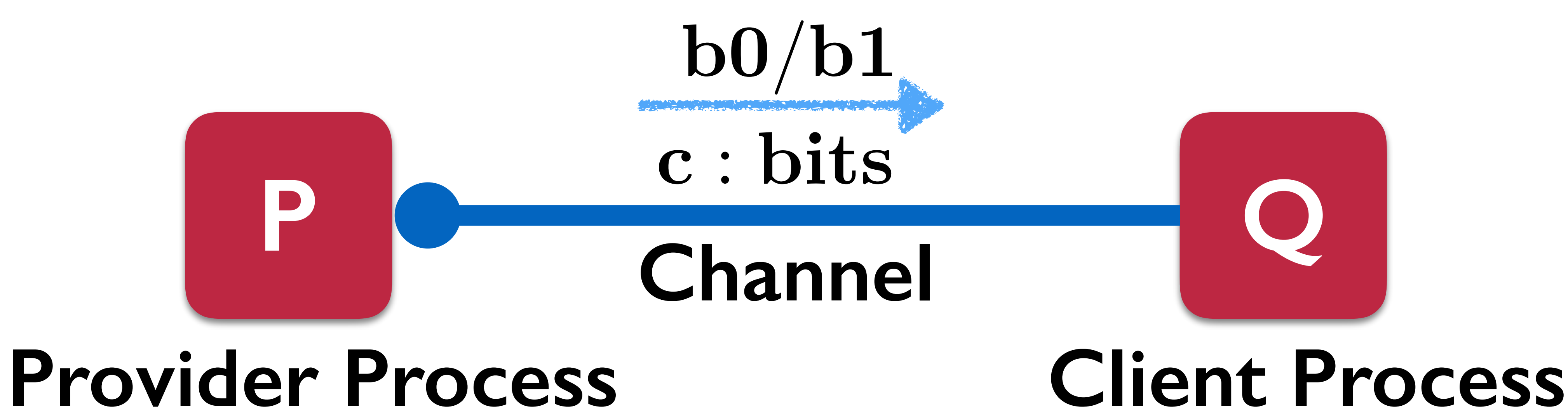

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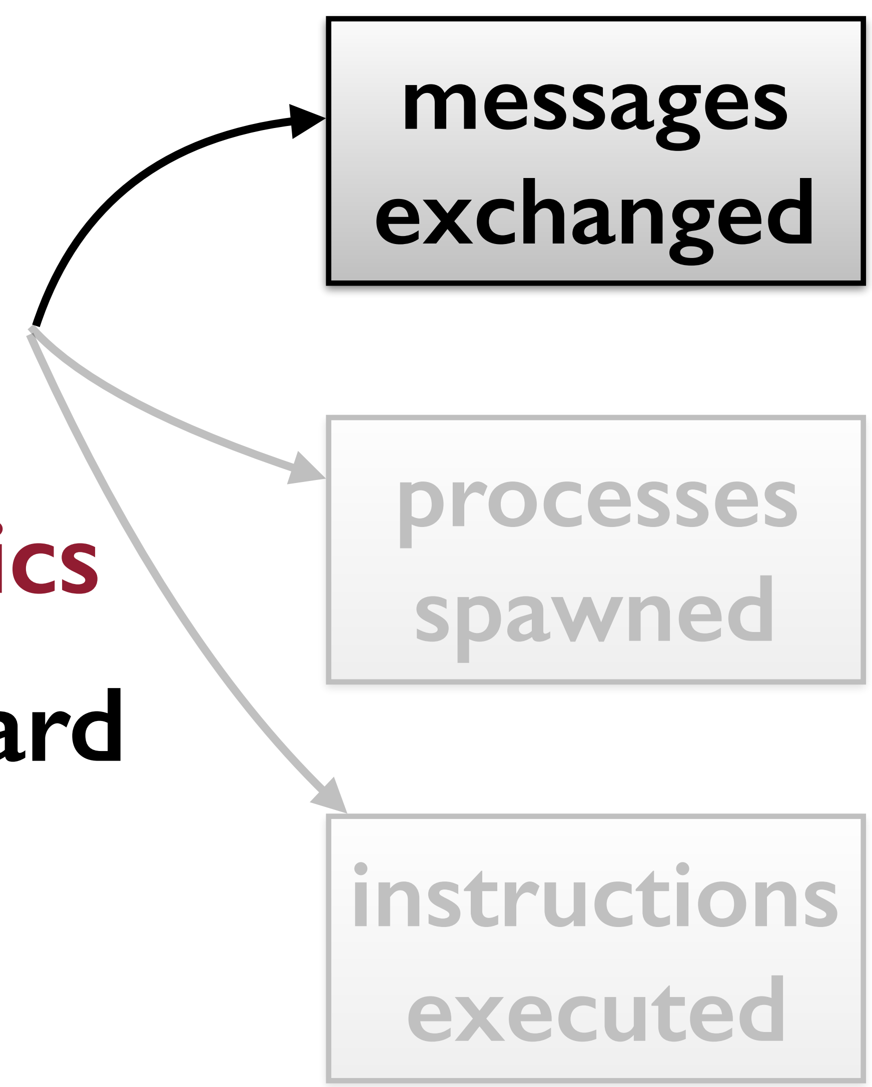
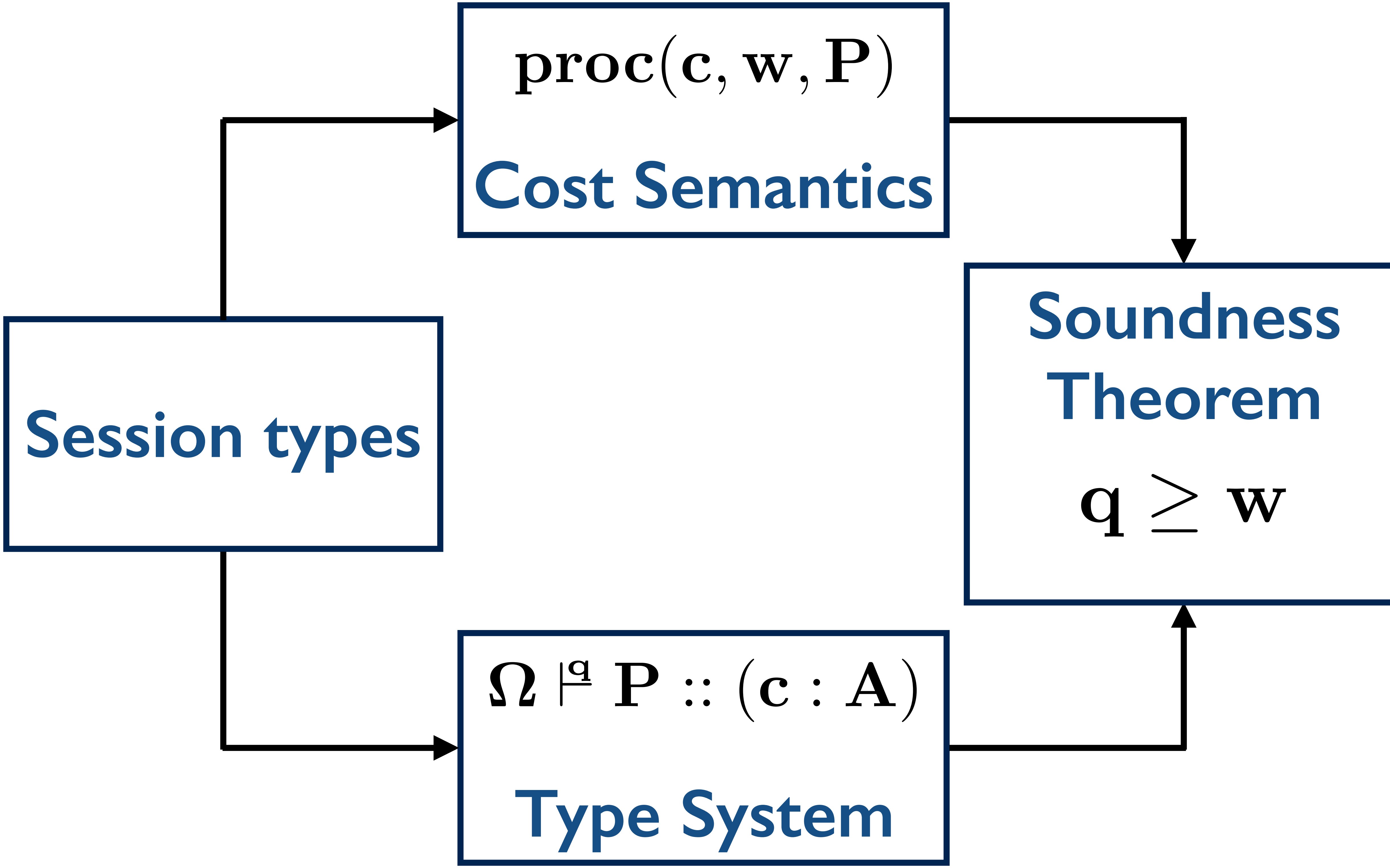


Overview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Cost Semantics
Session types
Soundness Theorem
$\mathbf{q \geq w}$
$\mathbf{\Omega \vDash^{q} P :: (c : A)}$
Type System

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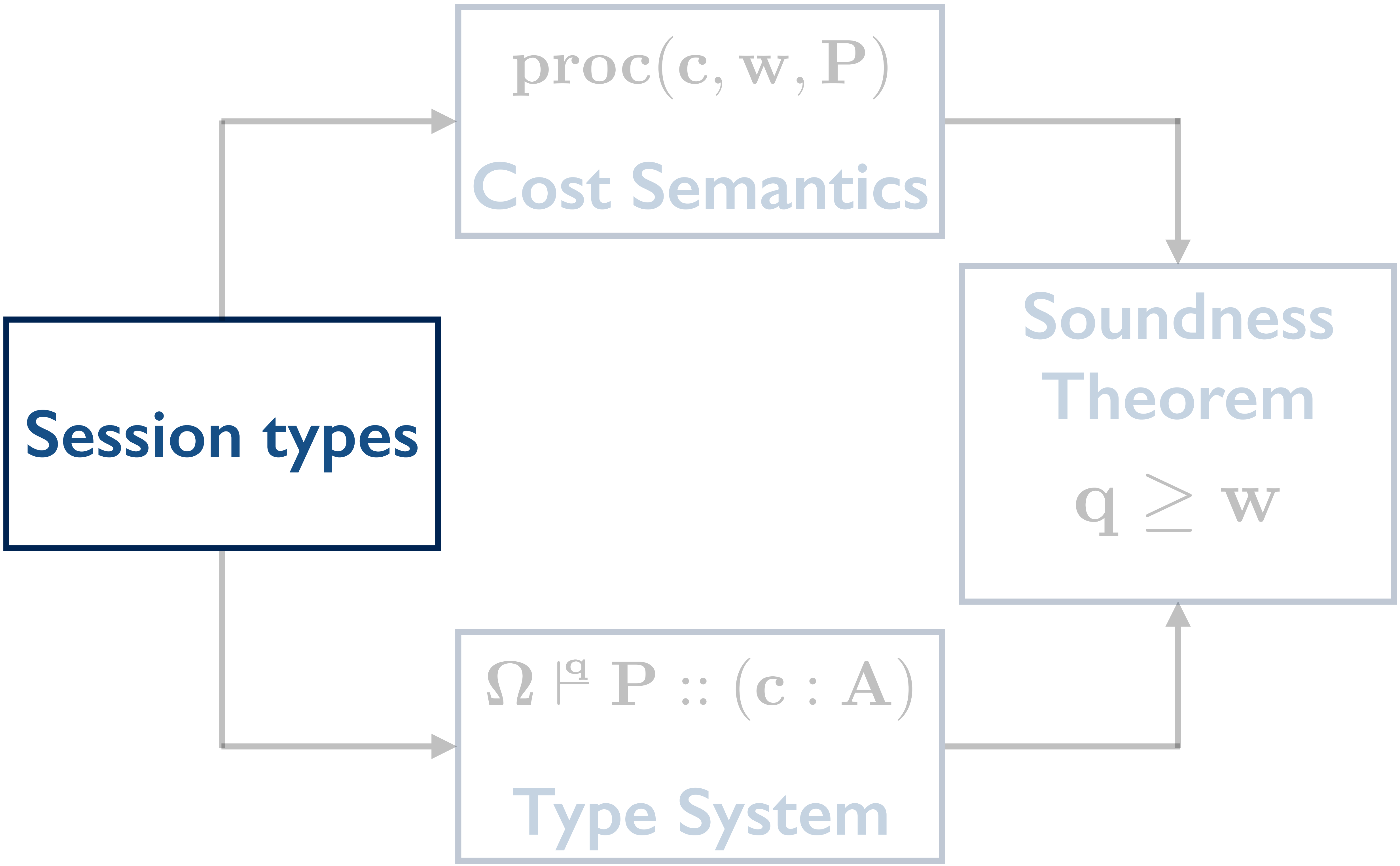

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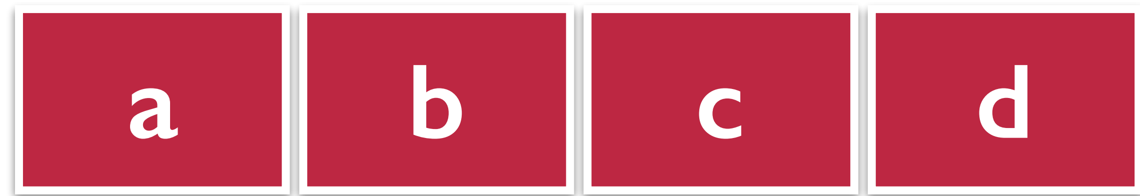
Type System

Example: Queues



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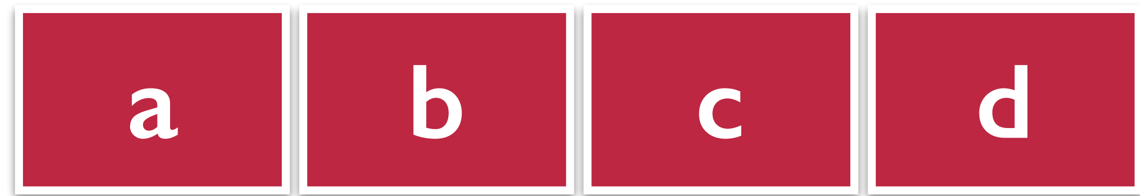

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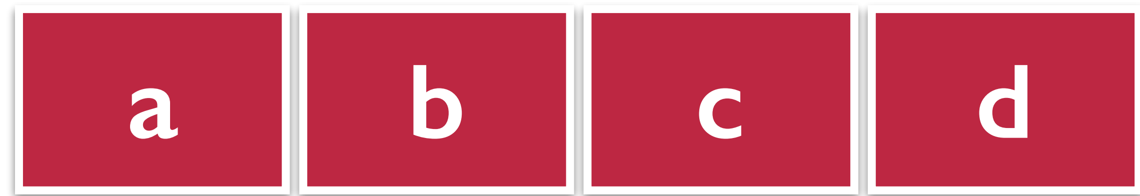

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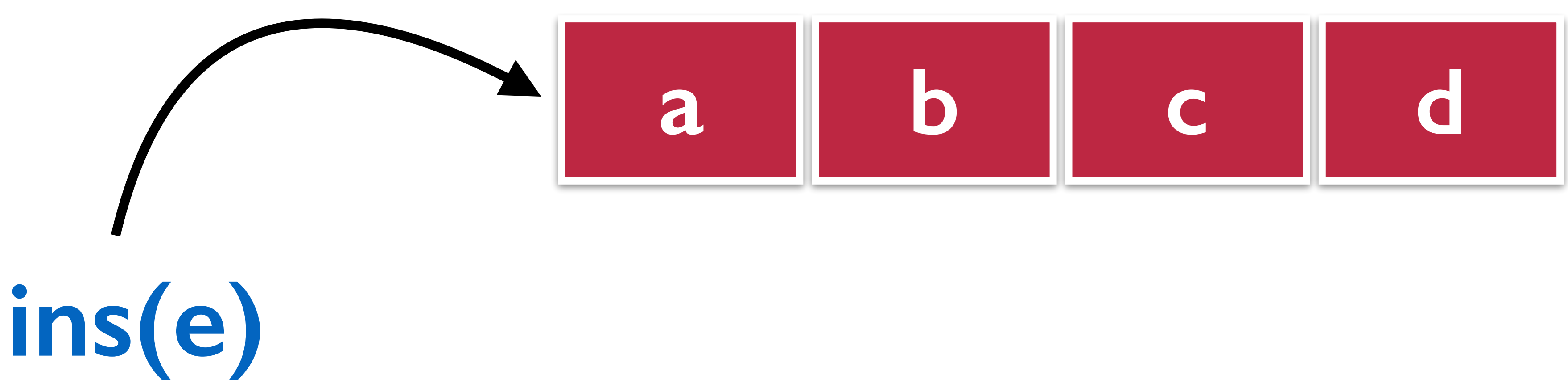

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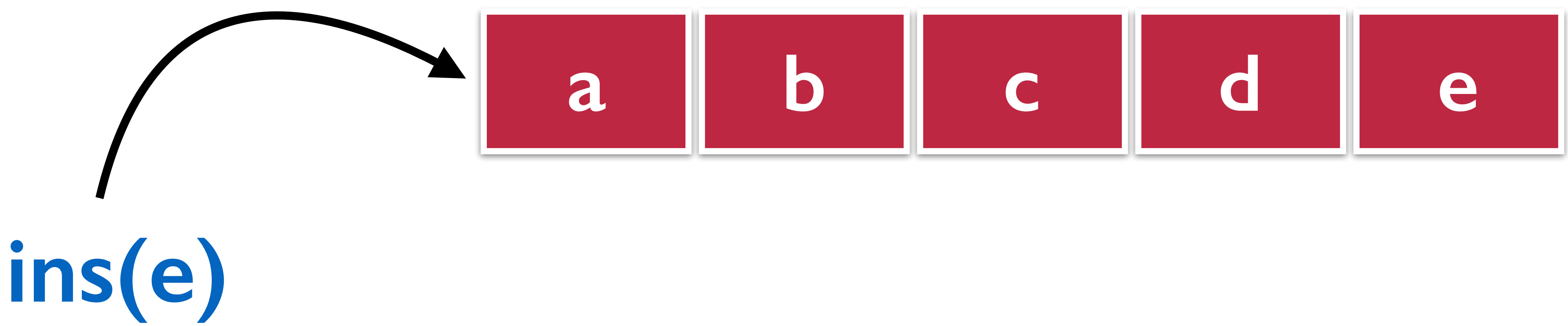

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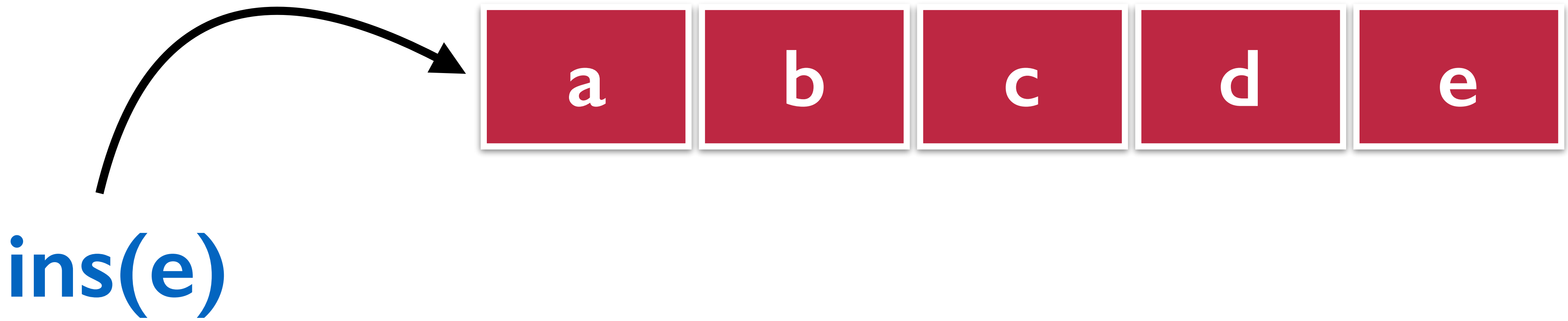

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send none if queue is empty

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terminate

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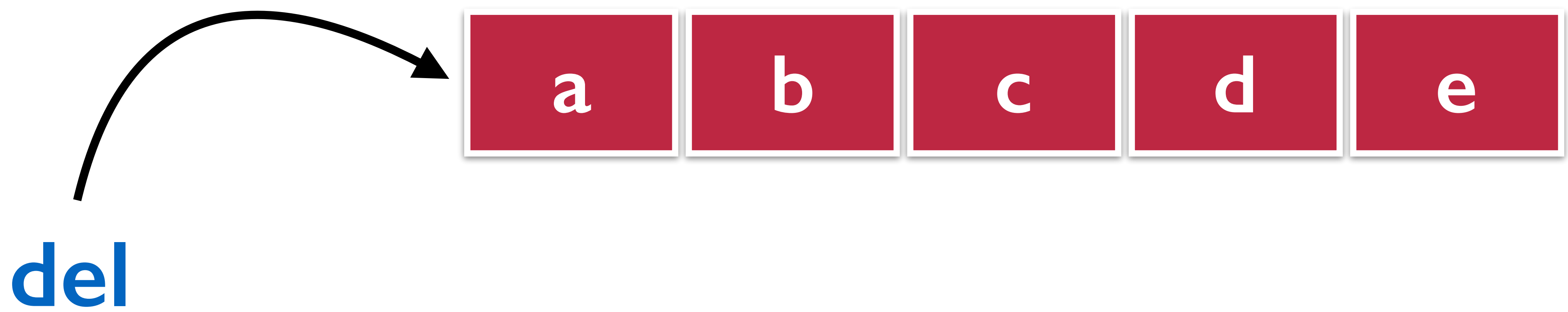

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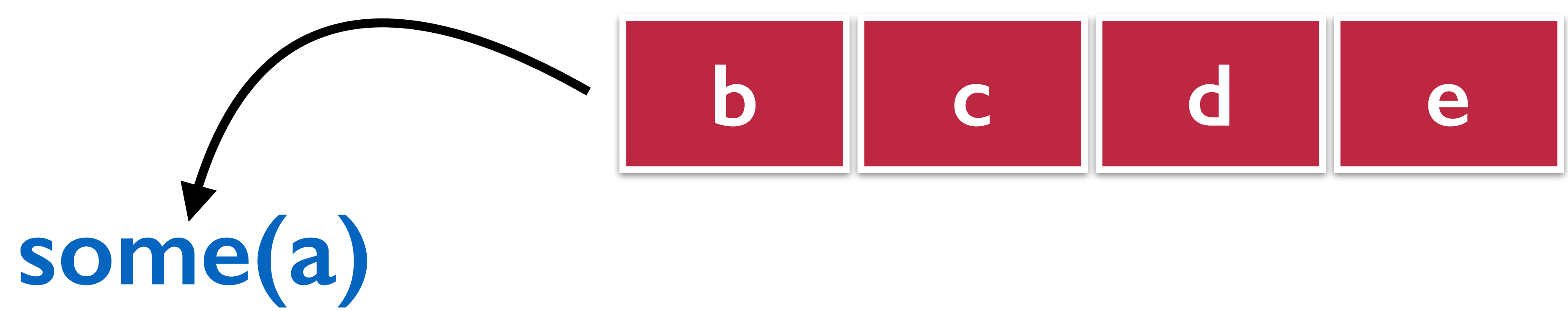

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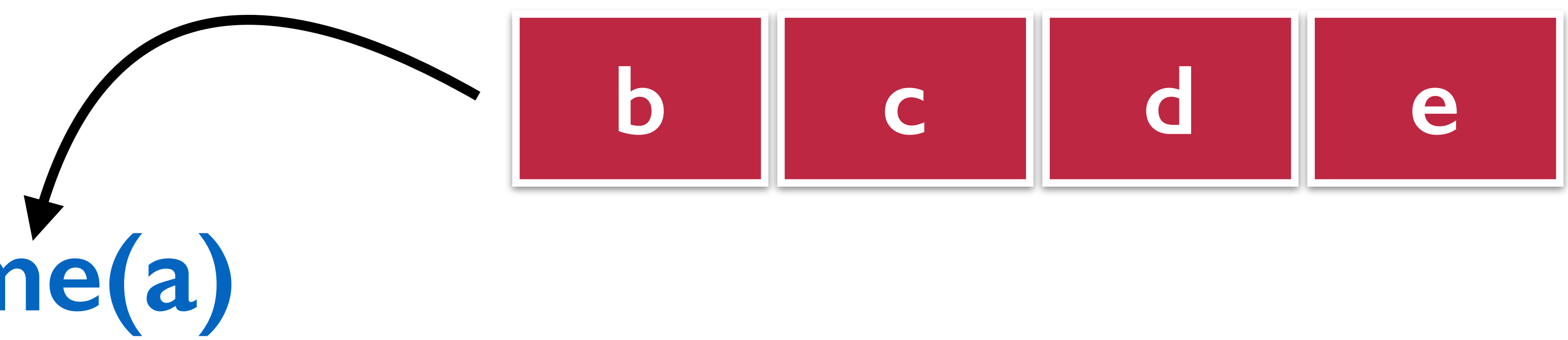

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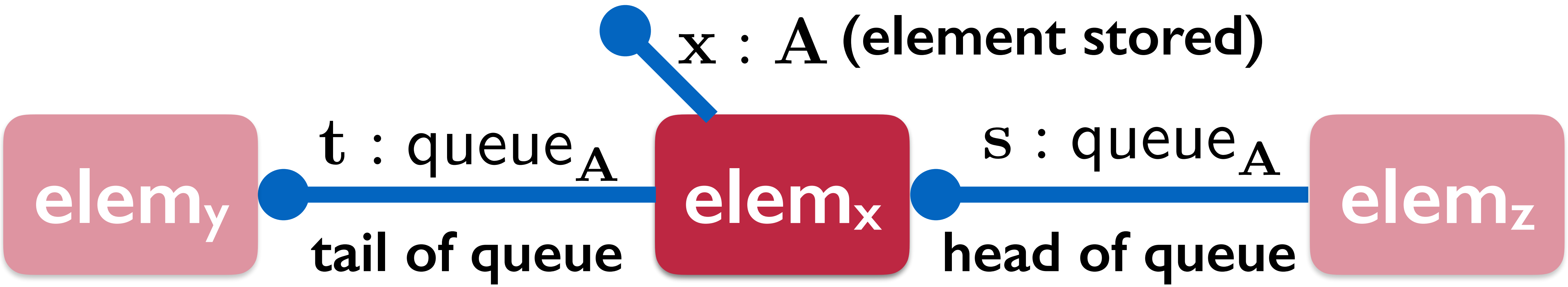
Session-typed Program



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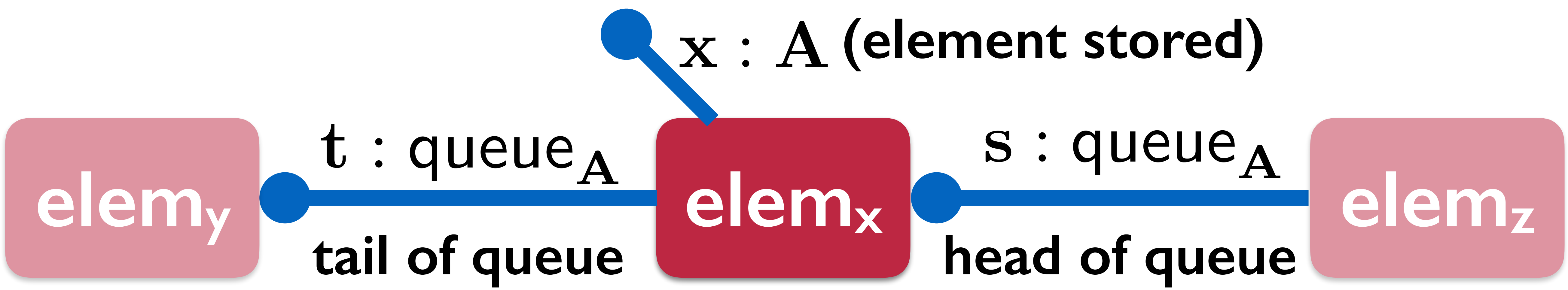

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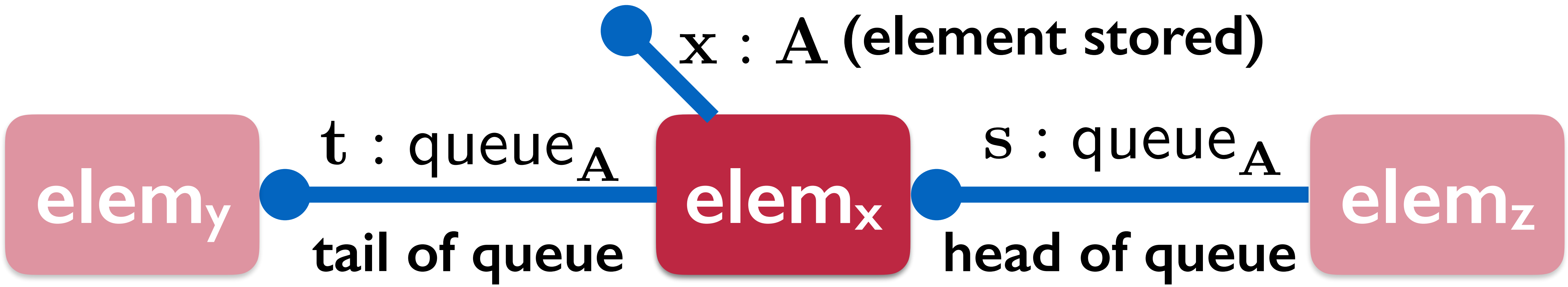

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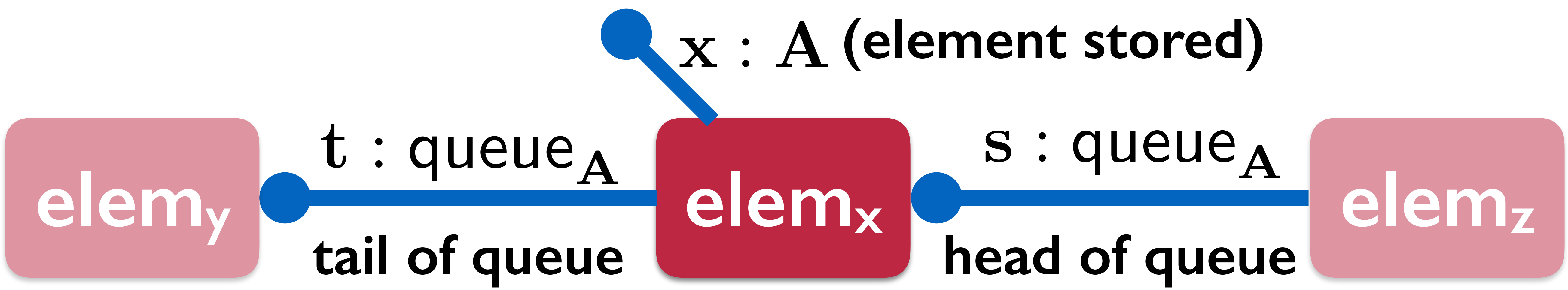

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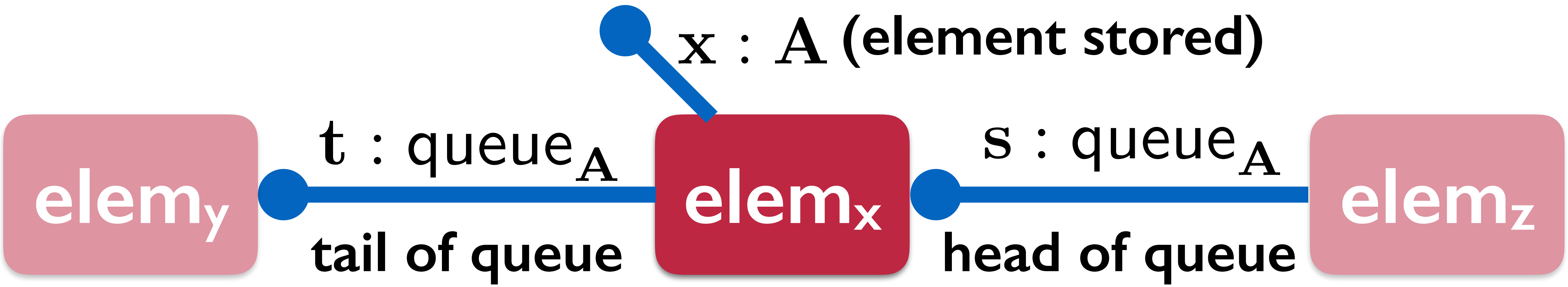

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send ‘some’, x

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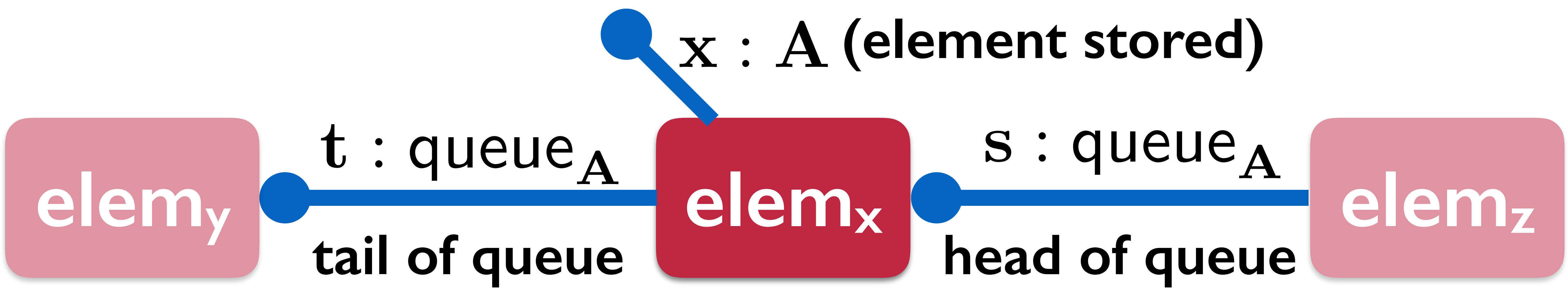

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\end{array}
$$

recv ‘ins’ and y

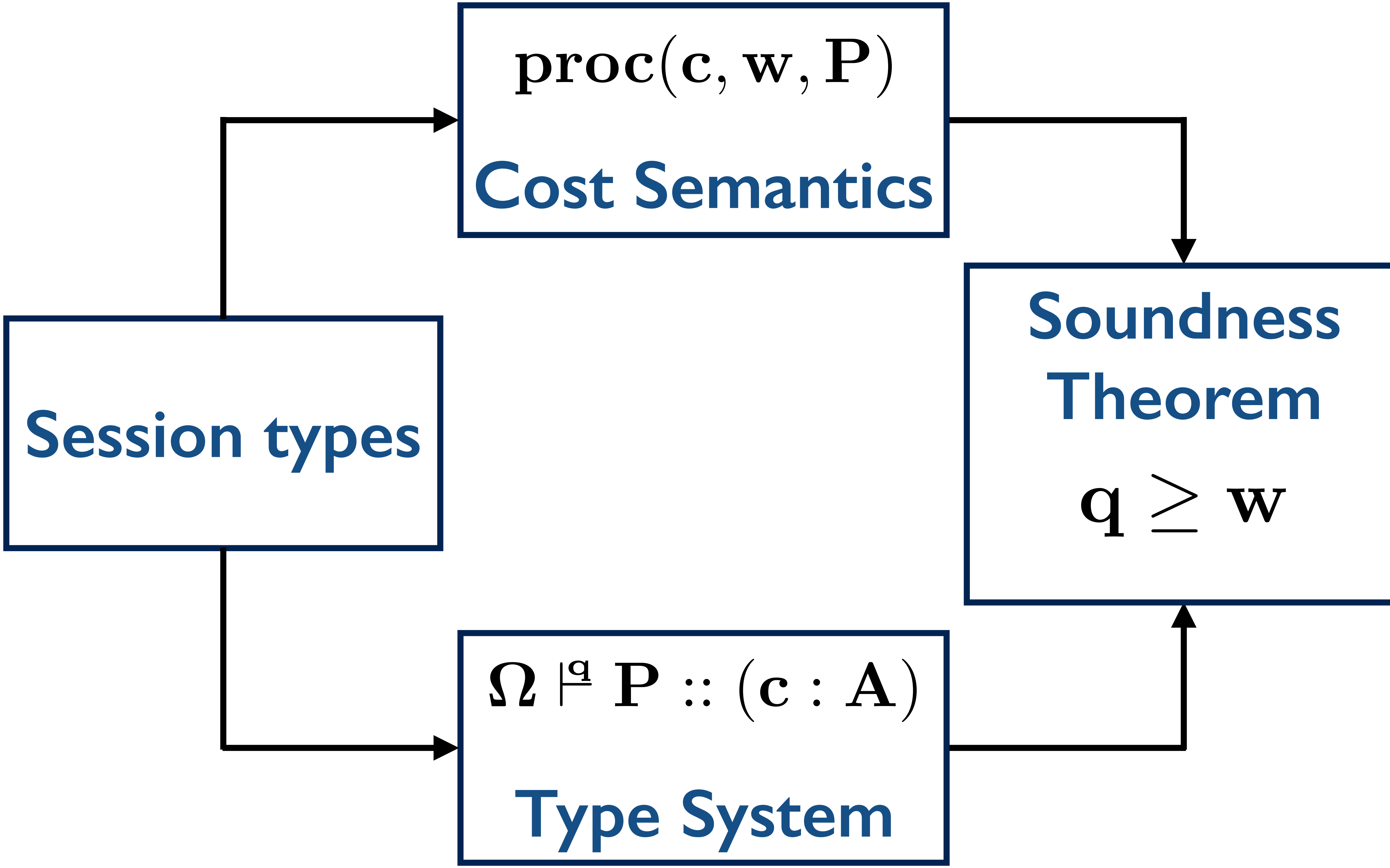
send ‘ins’ and y

recurse

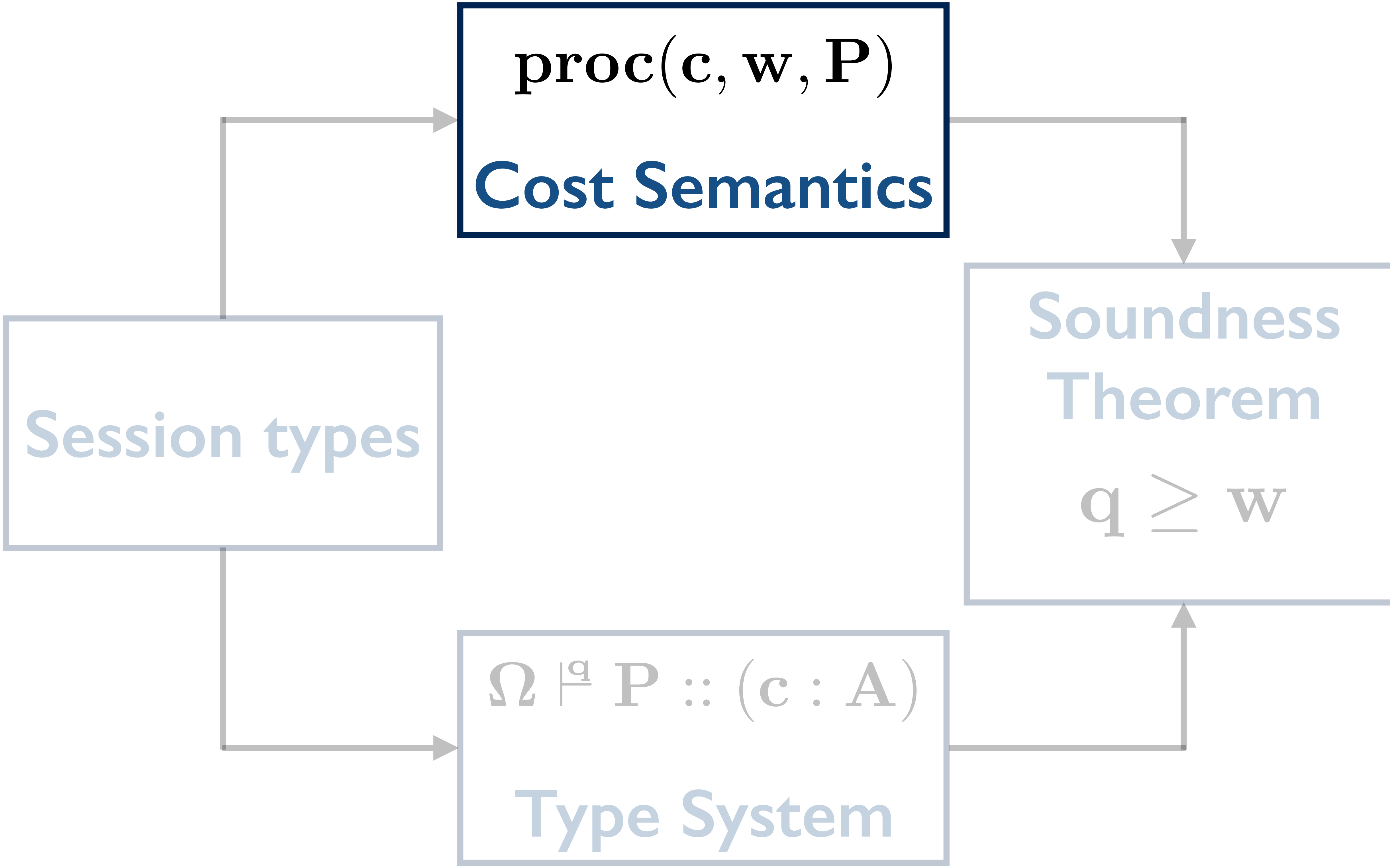
send ‘some’, x

terminate

Overview


$\mathbf{proc(c, w, P)}$
Cost Semantics
Session types
Soundness Theorem
$\mathbf{q \geq w}$
$\mathbf{\Omega \vdash^{q} P :: (c : A)}$
Type System

Overview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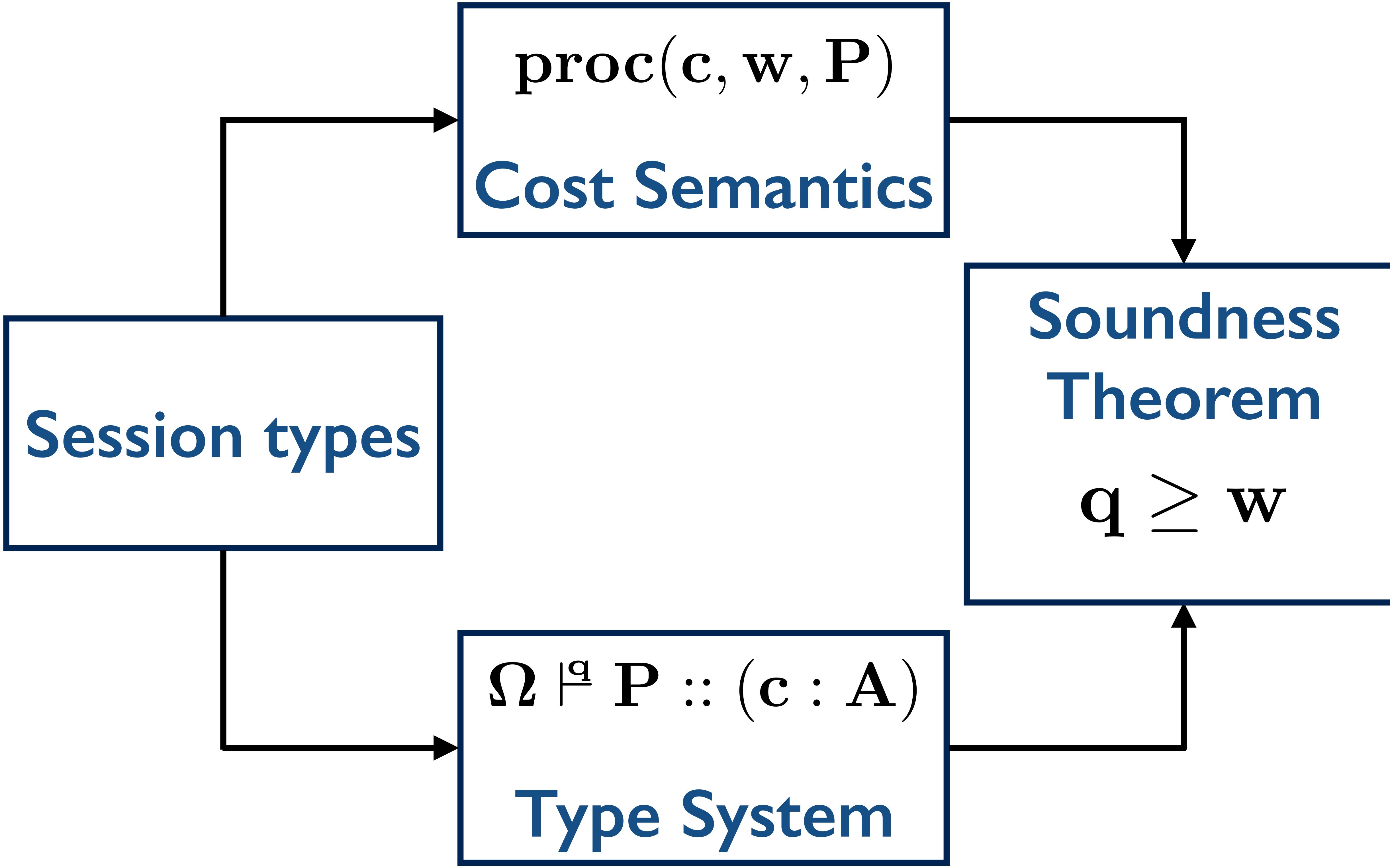
Cost Semantics

$$\mathbf{proc}(\mathbf{c}, \mathbf{w}, \mathbf{P})$$

Process P offering along channel c and has performed work w

- Standard semantics extended with local work counters w for each process
- Total work of system is sum of local counters w
- w is incremented every time process P performs some 'work' (this talk: whenever message is sent)

Overview



$\mathbf{proc(c, w, P)}$
Cost Semantics
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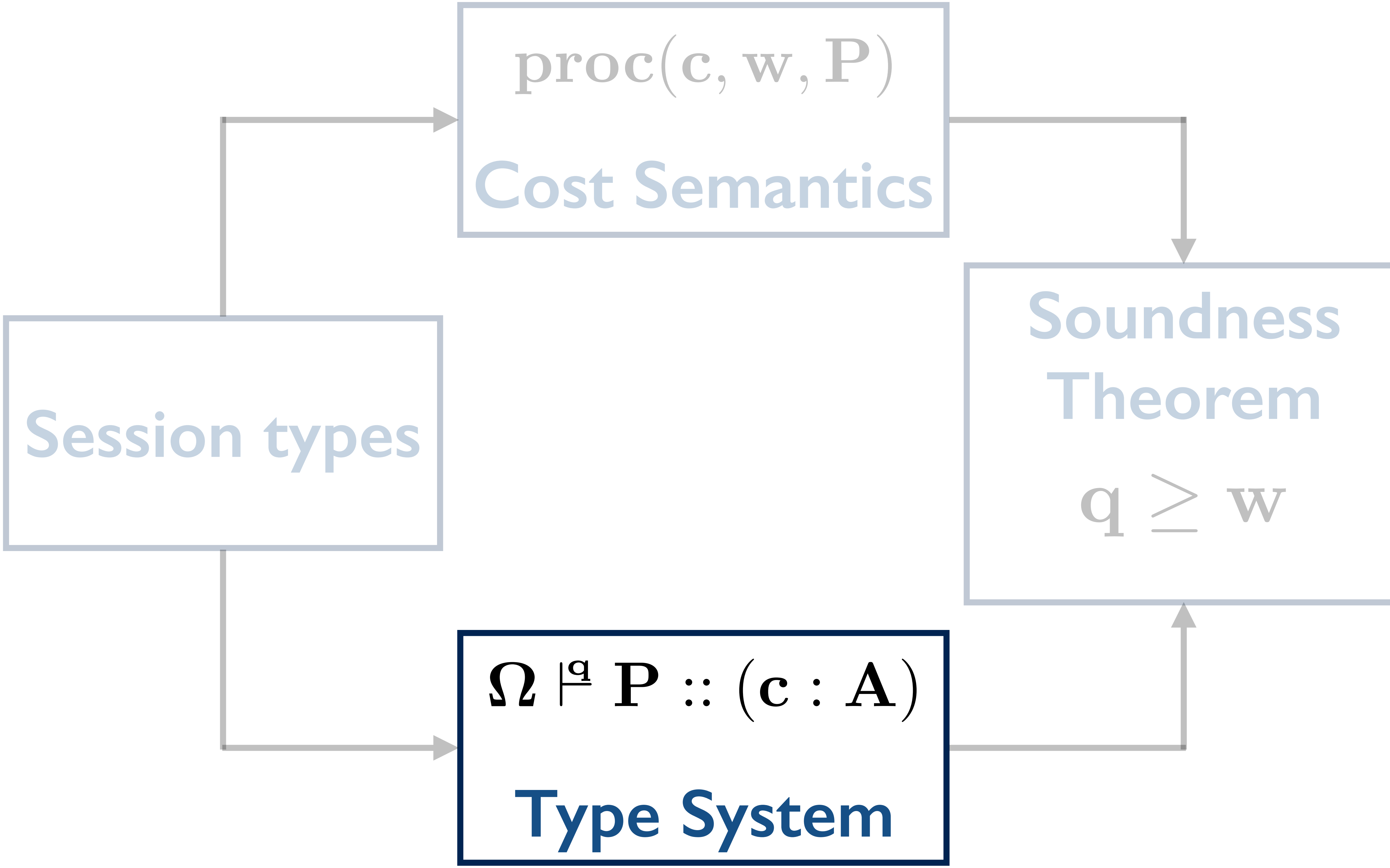

$\mathrm{proc}(c, w, P)$
Cost Semantics
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Soundness Theorem
$q \geq w$
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Type System

Type System

Based on Amortized Analysis!

- Store potential in each process
- Potential can be transferred via messages
- Potential is consumed to perform ‘work’

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$$\Omega \vdash^{q} P :: (c : A)$$

Process P offers along channel c,
acts as a client for channels in Ω
and storing potential q

Concurrent Queues

Count the messages!

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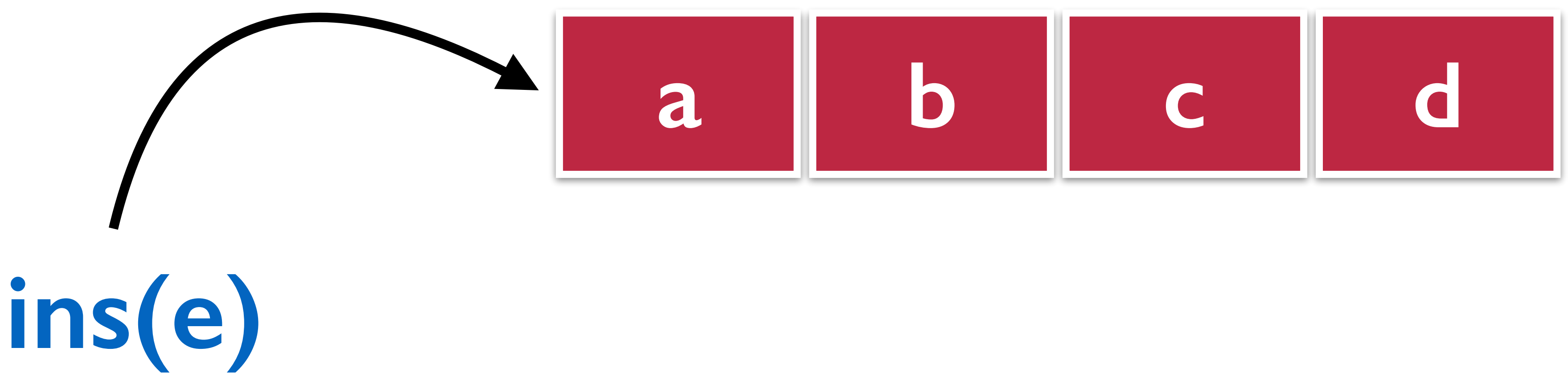

Concurrent Queues

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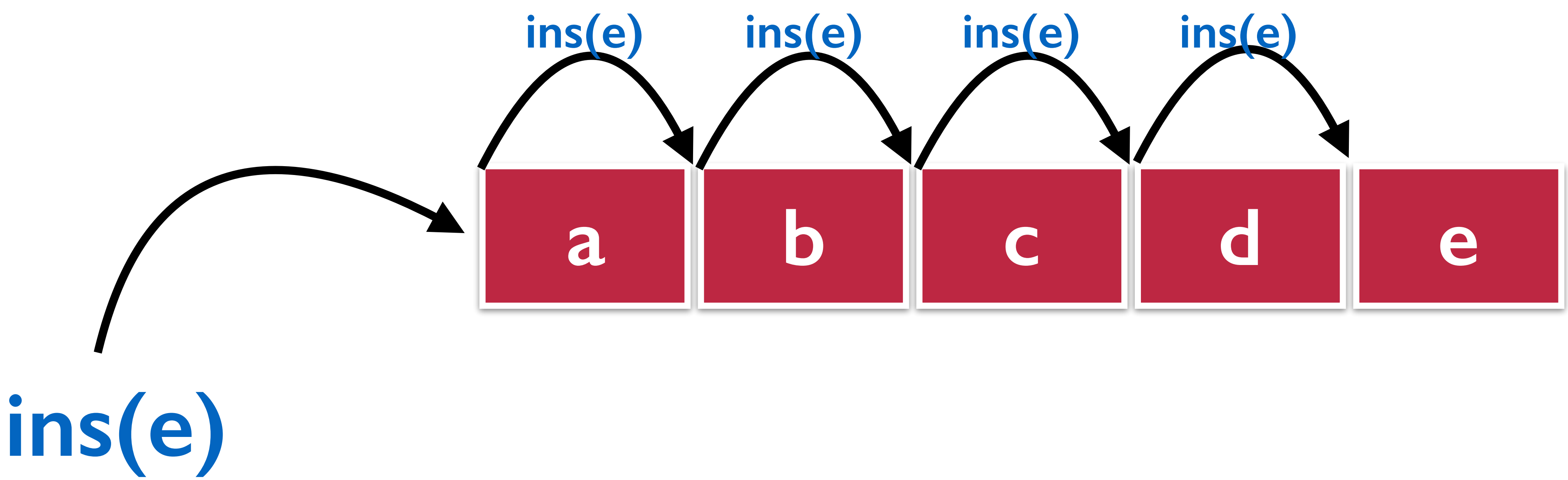

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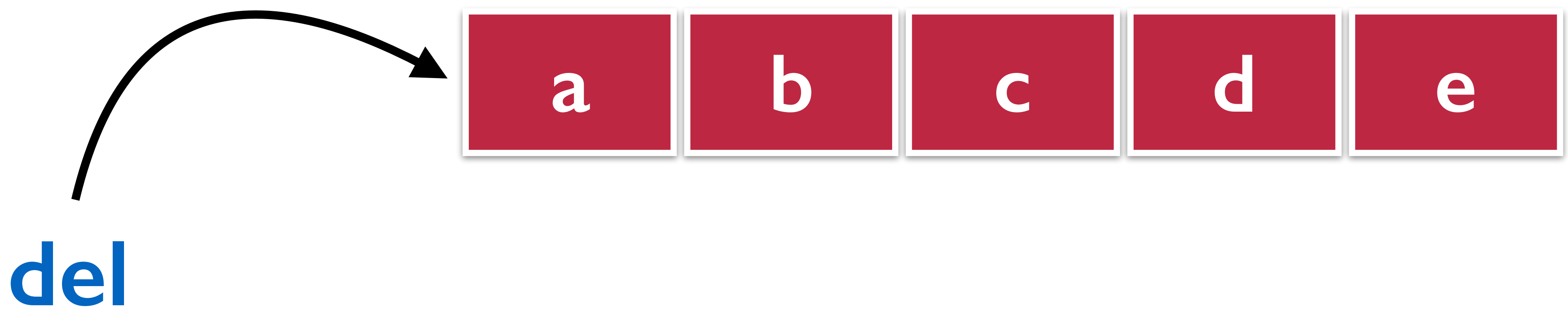

Concurrent Queues

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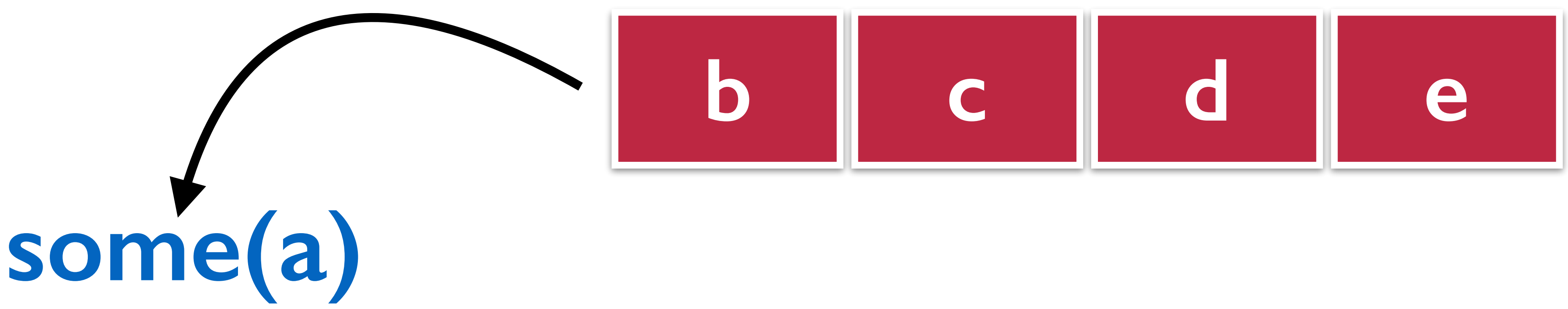

Concurrent Queues

Count the messages!

w_i = Work done to process insertion
= 2n (where n is the size of queue)
= ‘ins’ and ‘e’ travel to end of queue

w_d = Work done to process deletion
= 2 (sends back ‘some’ and ‘a’)

Type for Queues

$$\begin{aligned}
\mathsf{queue}_{\mathbf{A}}[\mathbf{n}] = \&\{&\mathsf{ins}^{\mathbf{2n}} : \mathbf{A} \multimap \mathsf{queue}_{\mathbf{A}}[\mathbf{n}+\mathbf{1}],\\
&\mathsf{del}^{\mathbf{2}} : \oplus\{\mathsf{none} : \mathbf{1},\\
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Resource-Aware Session Types

types augmented with potential information
sender pays potential with message
receiver gets potential with message

Type for Queues

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$$
\begin{array}{l}
(x : A)\ (t : \mathsf{queue}_A[n]) \vdash^{0} elem :: (s : \mathsf{queue}_A[n+1]) \\
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recv 2(n+1) units

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send 2n units

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send 2n units

cost of 2

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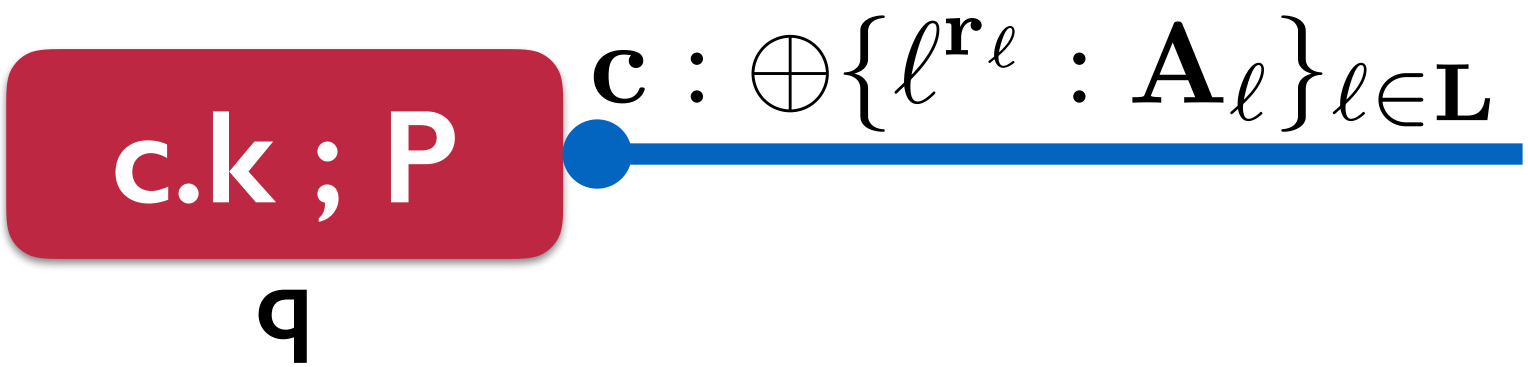
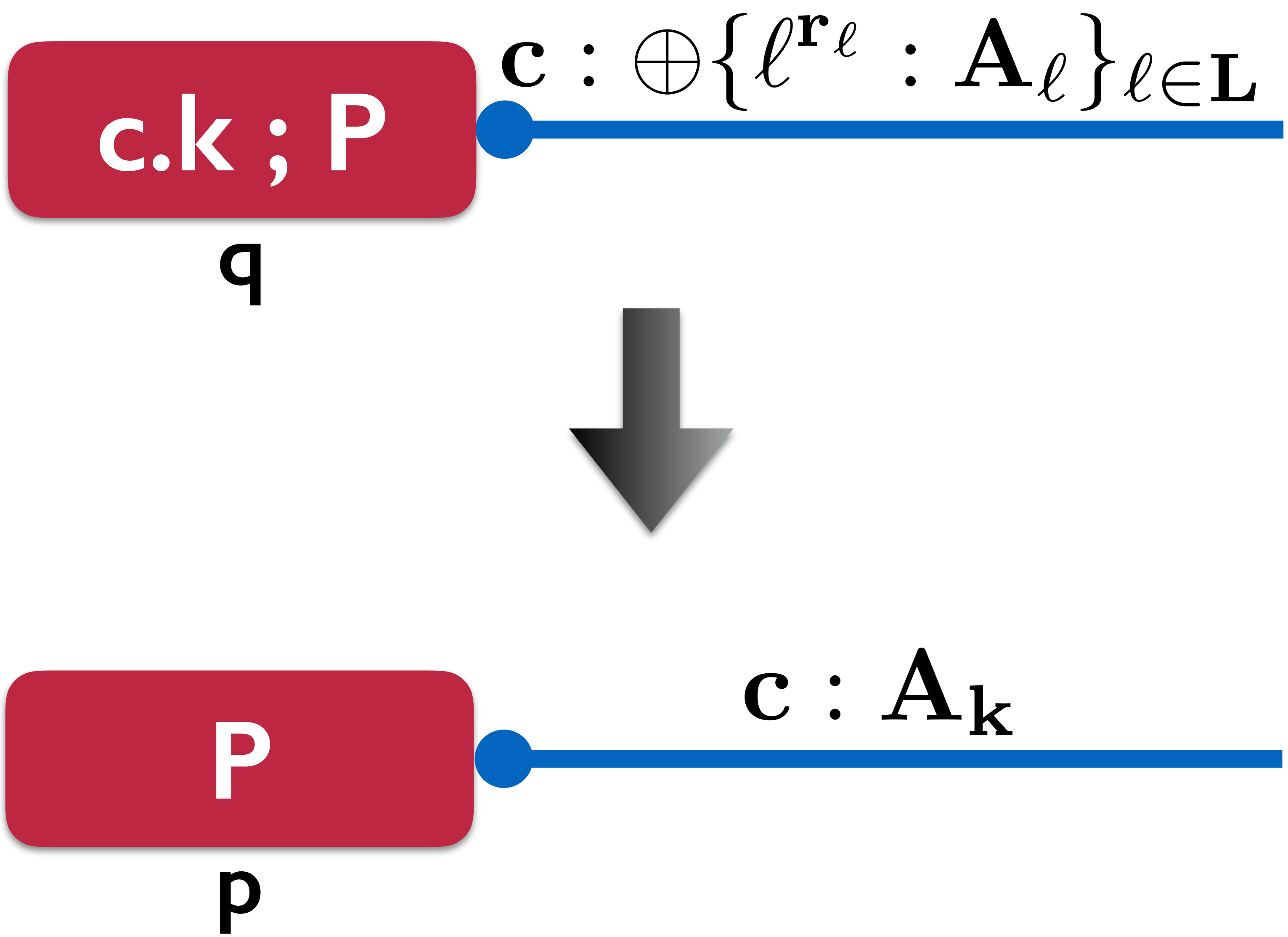
Rule: Sending a label



$\mathbf{\Omega} \models^{\mathbf{q}} \mathbf{P} :: (\mathbf{c} : \mathbf{A})$

Process P offers along channel c, acts as a client for channels in Ω and storing potential q

Rule: Sending a label



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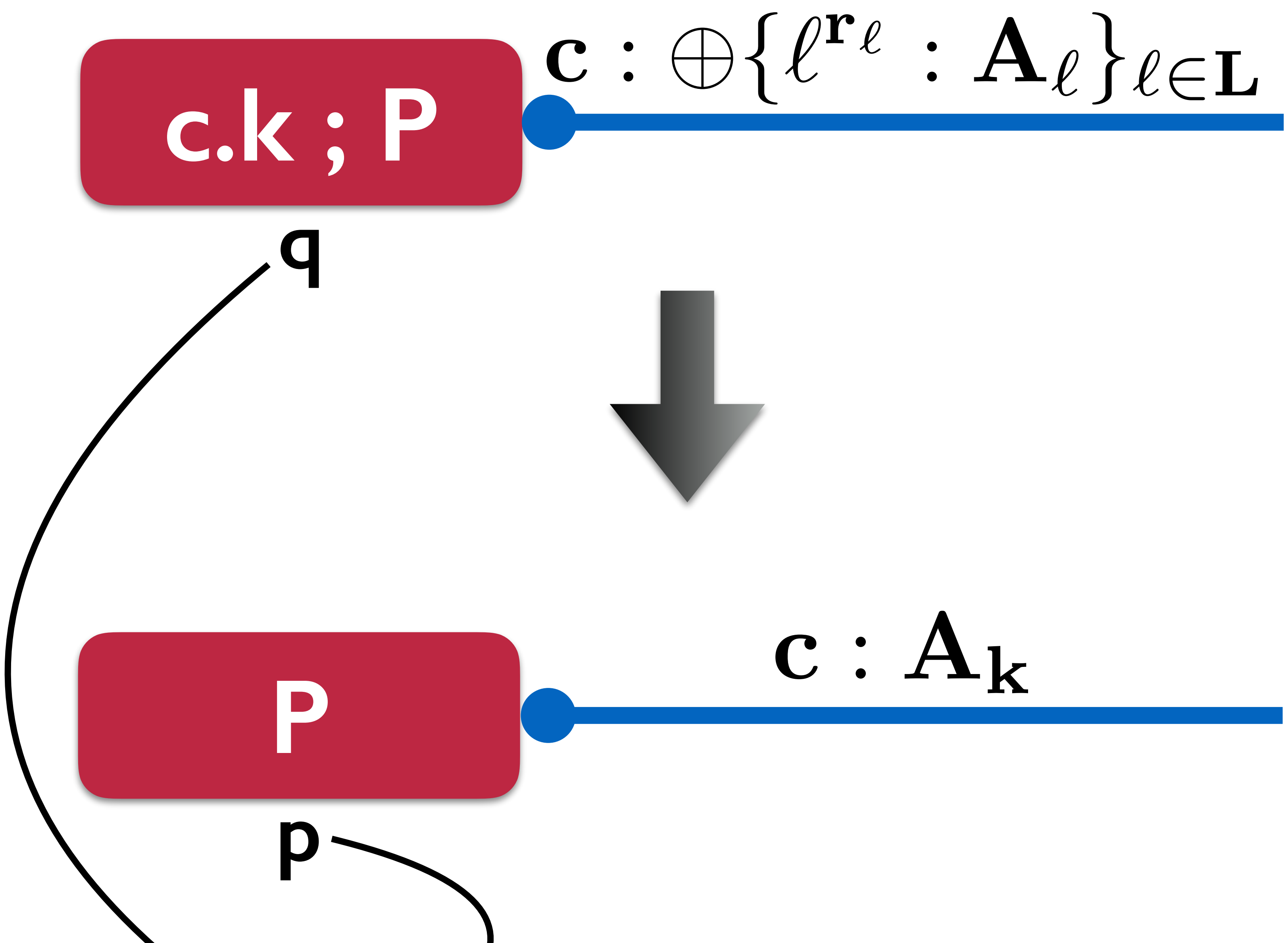

$\mathbf{\Omega} \overset{\mathbf{q}}{\vDash} \mathbf{P} :: (\mathbf{c} : \mathbf{A})$

Process P offers along channel c, acts as a client for channels in Ω and storing potential q

$$\frac{\mathbf{q} \geq \mathbf{p} + \mathbf{r_k} + 1 \qquad \mathbf{\Omega} \overset{\mathbf{p}}{\vDash} \mathbf{P} :: (\mathbf{c} : \mathbf{A_k})}{\mathbf{\Omega} \overset{\mathbf{q}}{\vDash} \mathbf{c.k} :: (\mathbf{c} : \oplus\{\ell^{\mathbf{r}_\ell} : \mathbf{A}_\ell\}_{\ell \in \mathbf{L}})}$$

Features of Type System

- **Flexible:** supports counting of different resources (e.g. messages exchanged, processes spawned, etc.) by being parametric in cost model
- **Compositional:** types describe individual processes, not just whole programs
- **Precise:** potential upper bounds work accurately
- **Conservative:** strict extension of type system
- **General:** works on most standard examples
- **Automatic:** future work

Examples!

Binary Counters

$$\mathsf{ctr}[\mathbf{n}] = \&\{\mathsf{inc}^{\mathbf{1}} : \mathsf{ctr}[\mathbf{n}+\mathbf{1}],$$
$$\mathsf{val}^{\mathbf{2}\lceil\mathbf{log}(\mathbf{n}+\mathbf{1})\rceil+\mathbf{2}} : \mathsf{bits}\}$$

- **Increment:**
  - **requires one unit of potential**
  - **uses amortized analysis!**
- **Value:**
  - **requires logarithmic potential**
  - **precise work bound**

Stacks vs Queues

$$\begin{aligned}\mathsf{stack}_{\mathbf{A}}[\mathbf{n}] = \&\{&\mathsf{ins}^{\mathbf{0}} : \mathbf{A} \multimap \mathsf{stack}_{\mathbf{A}}[\mathbf{n}+\mathbf{1}],\\ &\mathsf{del}^{\mathbf{2}} : \oplus\{\mathsf{none} : \mathbf{1},\\ &\qquad\qquad \mathsf{some} : \mathbf{A} \otimes \mathsf{stack}_{\mathbf{A}}[\mathbf{n}-\mathbf{1}]\}\}\end{aligned}$$

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Which one's more efficient?

Stacks vs Queues

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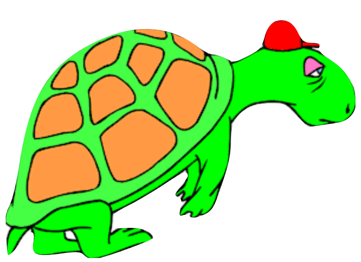

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Contributions

Type System for Work Analysis

based on amortized analysis

types augmented with potential information

work is upper bounded by potential

Flexible, Compositional, Precise, Conservative, General, Automatic

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Cost Semantics

Examples

stacks, queues, binary counters

efficiency comparison

list examples: append, map, fold