Parallel Complexity Analysis with Temporal Session Types

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Frank Pfenning

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Computer Science Department

What is Parallel Complexity?

a.k.a. Span

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Data Dependencies	Wait for Messages	Data Races Shared Memory

Why Parallel Complexity?

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Complexity of Parallel Algorithms

Blelloch (Comm. ACM '96)

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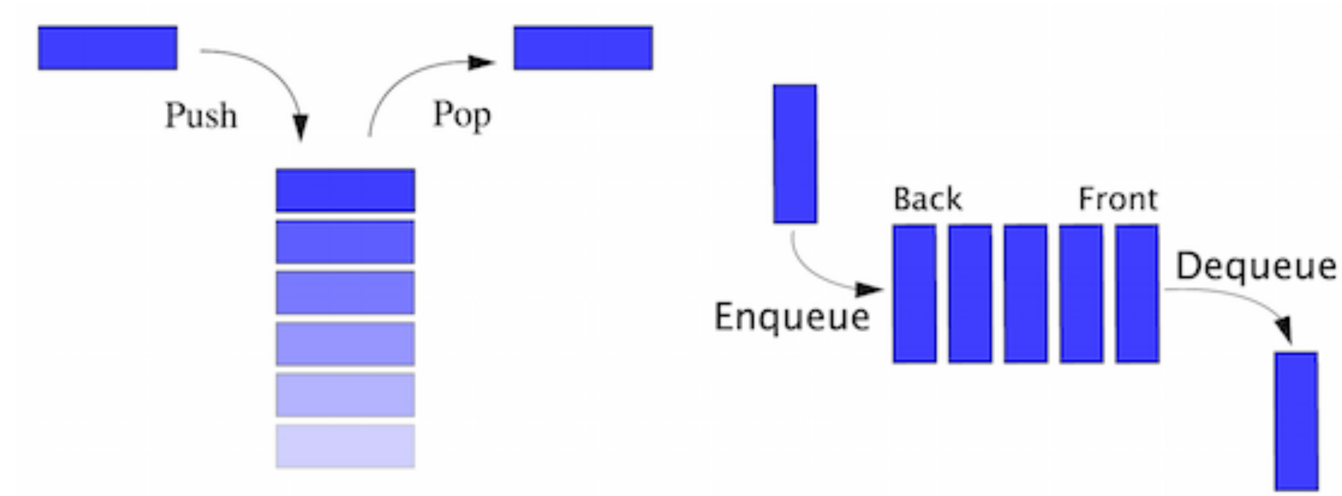

Response Time of Concurrent Data Structures

Ellen and Brown (PODC '16)

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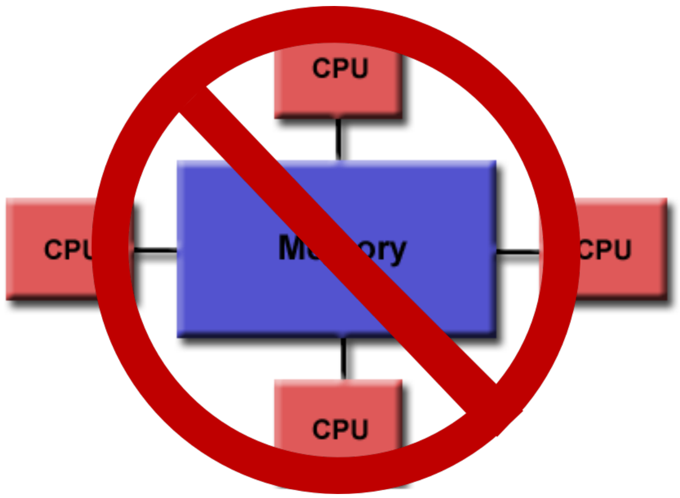

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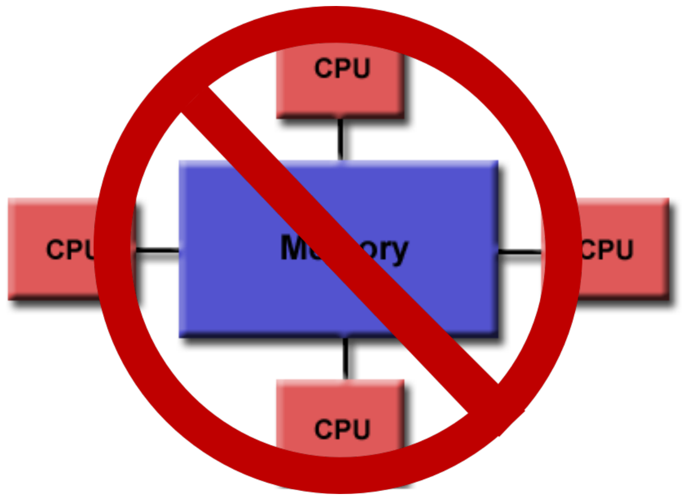

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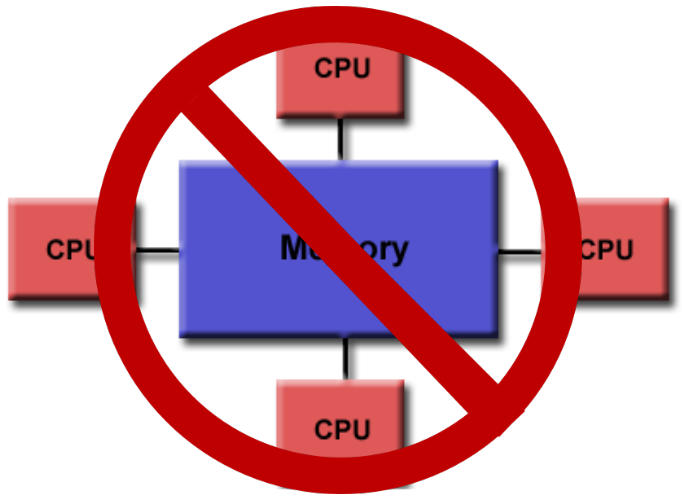

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Deadlock Freedom

What are Session Types?

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- Communication via typed bi-directional channels
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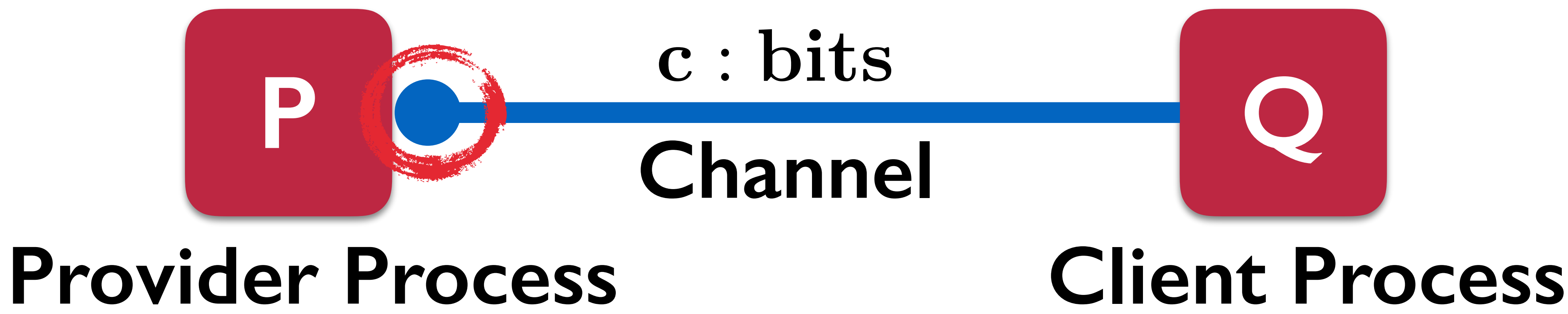

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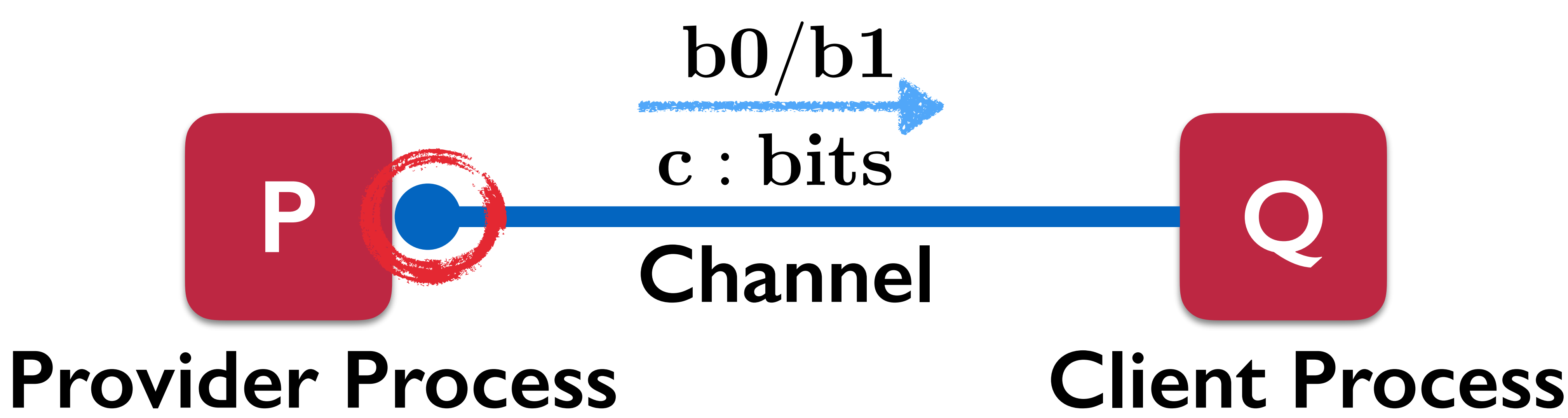

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Type system to analyze timings of message exchanges of session-typed programs

- types define the *timing* of message exchanges
- provides *precision* and *flexibility*
- proved *sound* w.r.t. *cost semantics* tracking time
- *conservative* extension to typical session type system
- applies to all *standard* session types examples
- can be *parameterized* to count resource of interest

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- ***Cost model*** **assigns a time cost to each operation**

$\mathcal{R}$ cost model

Unit delay after each receive

$\mathcal{RS}$ cost model

Unit delay after each receive and send

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$\mathcal{R}$ cost model
Unit delay after each receive

$\mathcal{RS}$ cost model
Unit delay after each receive and send

- Expressed by inserting appropriate delays in the source code, only the delays cost time
- Programmer specifies cost model, compiler automatically inserts delays for type checking

$$\Omega \vdash \mathrm{P} :: (\mathrm{x} : \mathrm{S})$$

Definition of the Types

Track Message Rates

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Compute output rate given input rate

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timing of messages ⇔ Parallel Complexity

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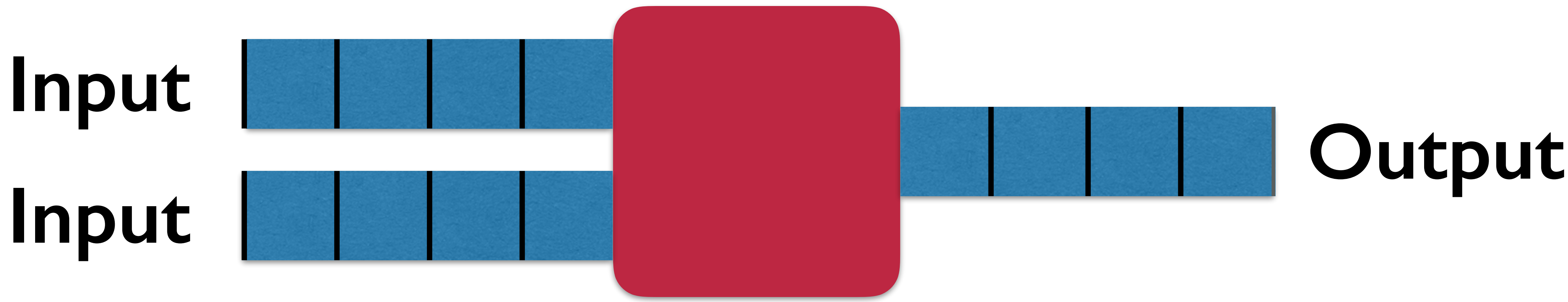

Compute output rate given input rate

timing of messages ⇔ Parallel Complexity

Necessary:
need exact input/output rate to ensure compositionality

Sufficient:
span can be thought as timing of final message

Example: Bit Streams

$$\mathbf{bits} = \oplus\{\mathbf{b0} : \mathbf{bits}, \mathbf{b1} : \mathbf{bits}, \$: \mathbf{1}\}$$

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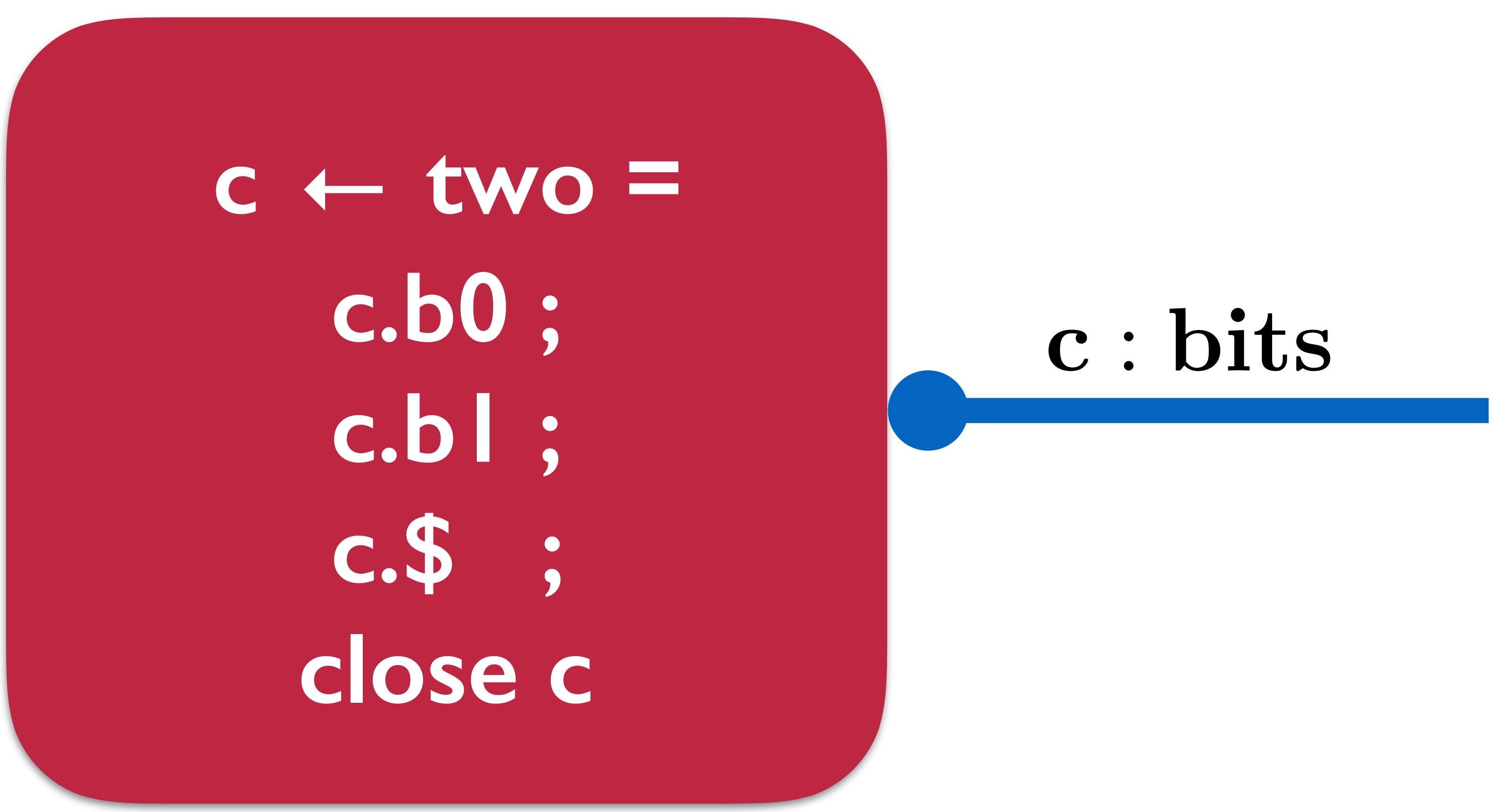

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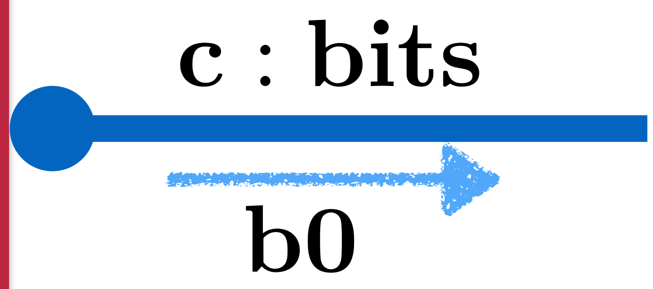

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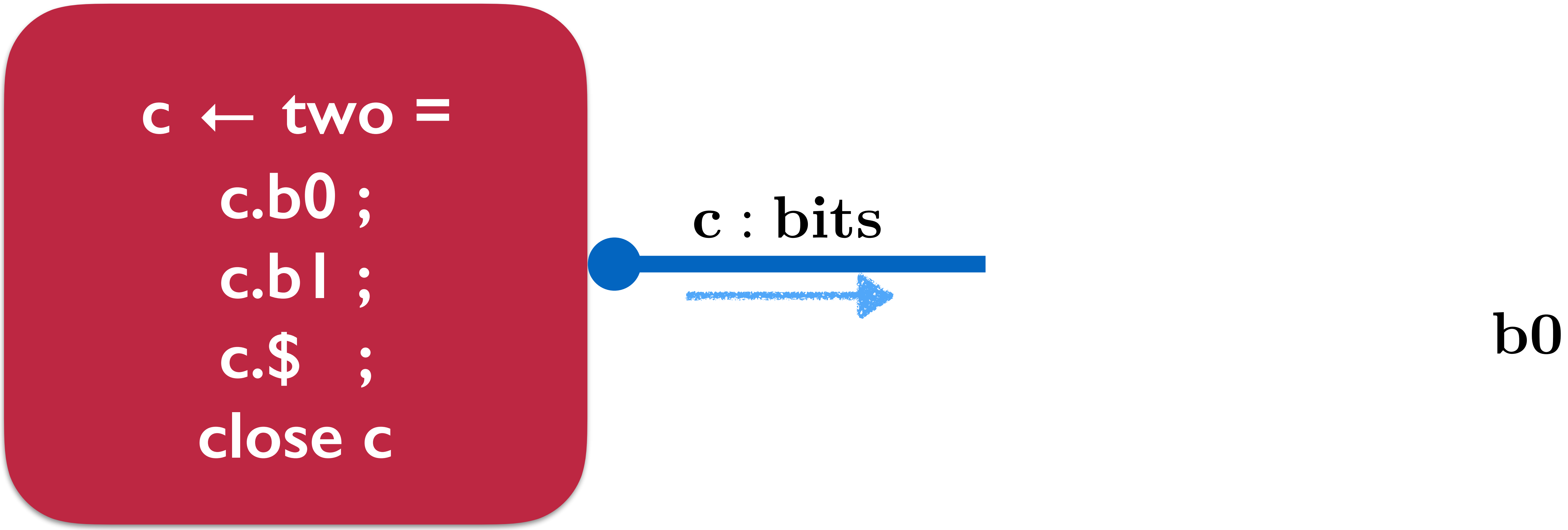

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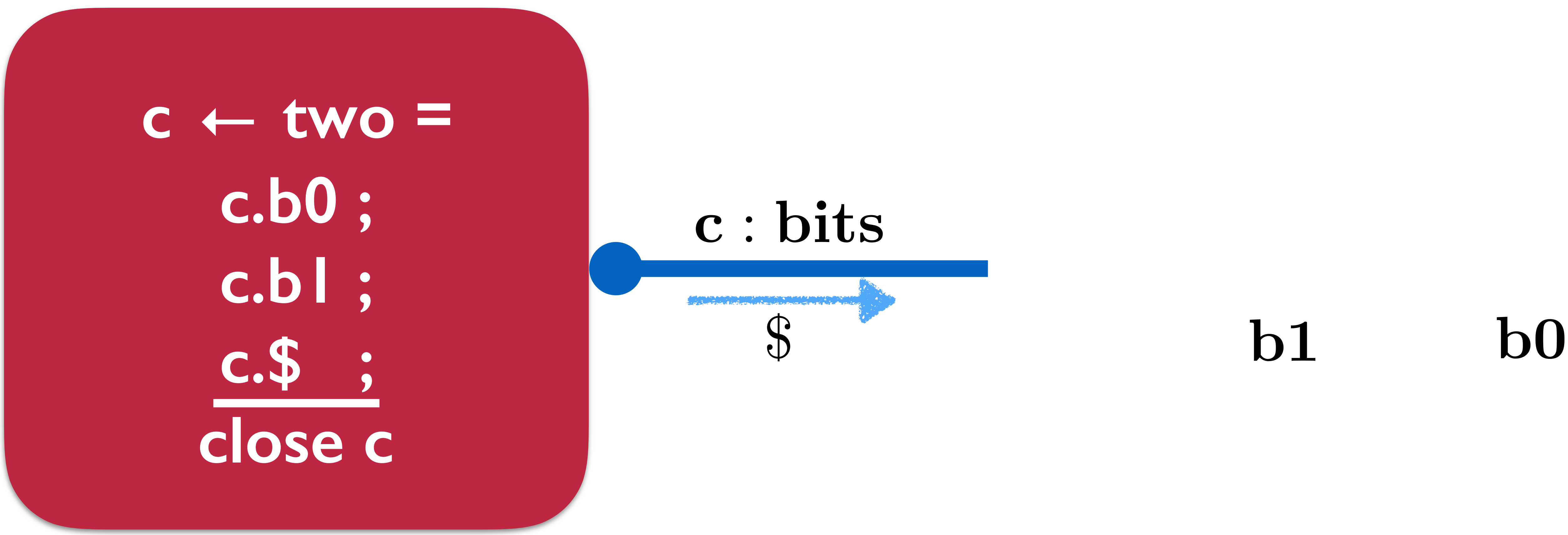

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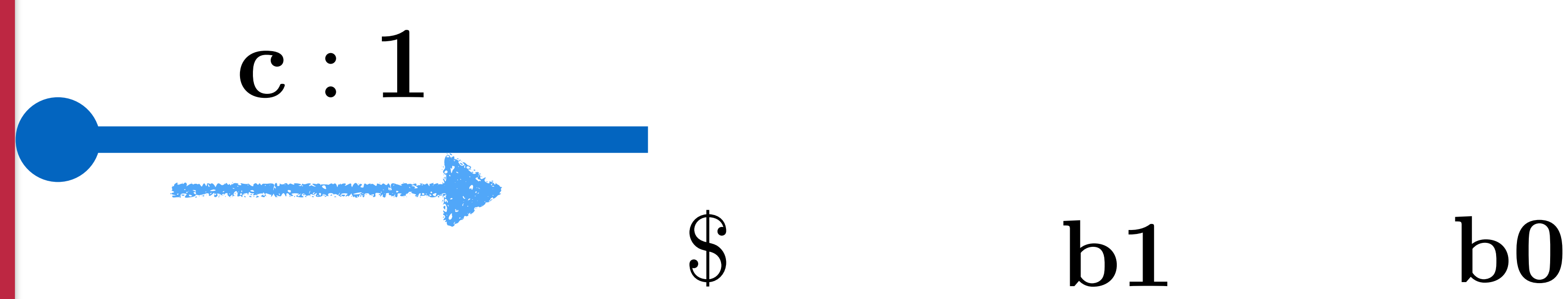

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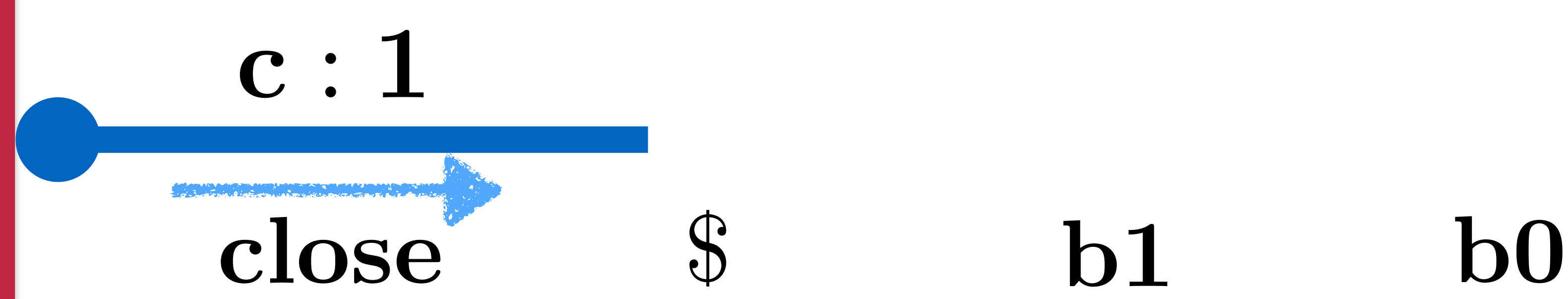

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$$\cdot \vdash \text{two} :: (c : \text{bits})$$

Timing Information?

```
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Sending a message causes unit delay

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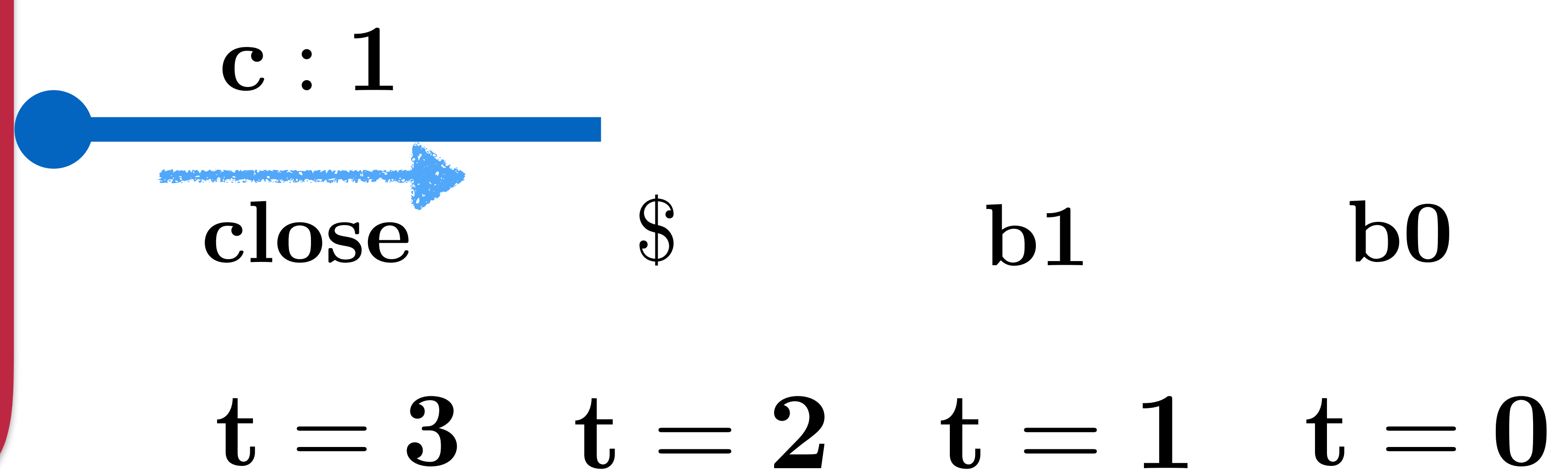

Enforcing Time in the Type

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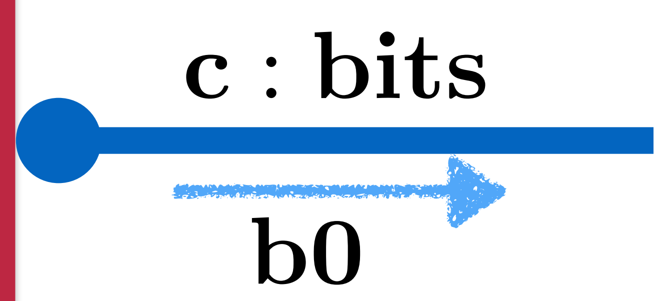

$$\mathbf{t = 0}$$

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$\mathbf{b0}$

$\mathbf{t = 0}$

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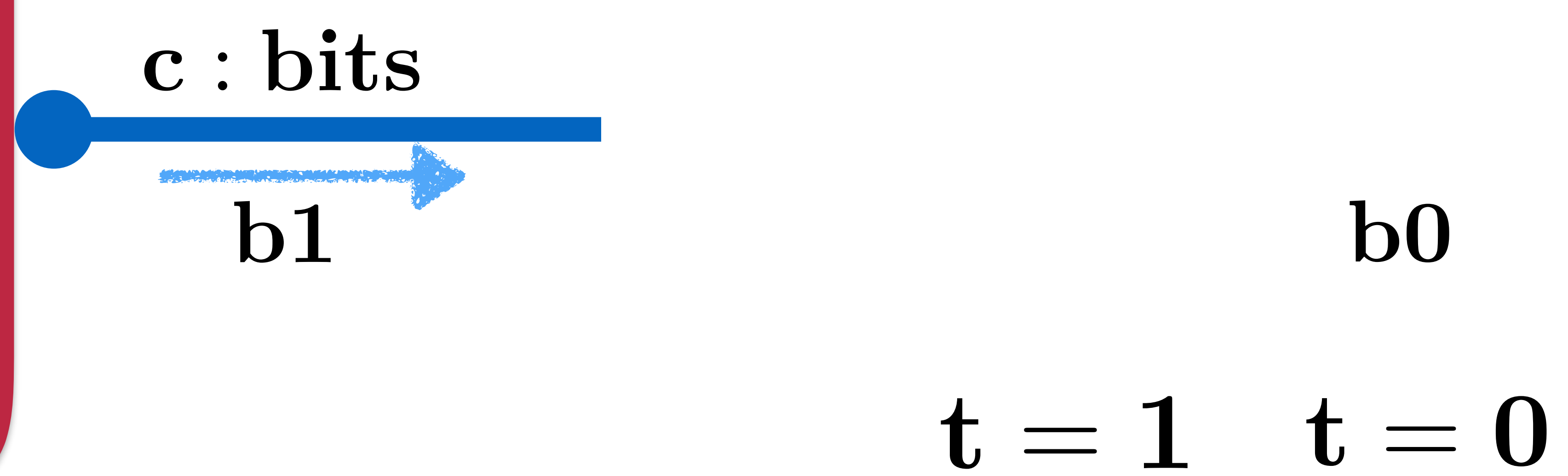

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b1 b0

$\mathbf{t = 1}$ $\mathbf{t = 0}$

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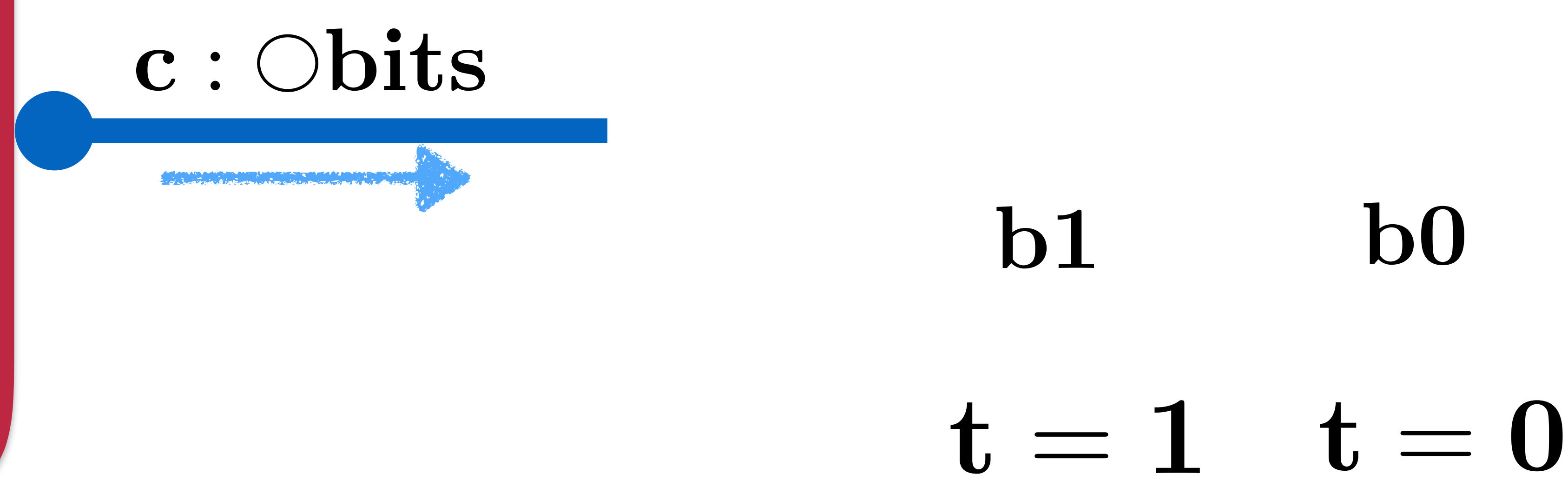

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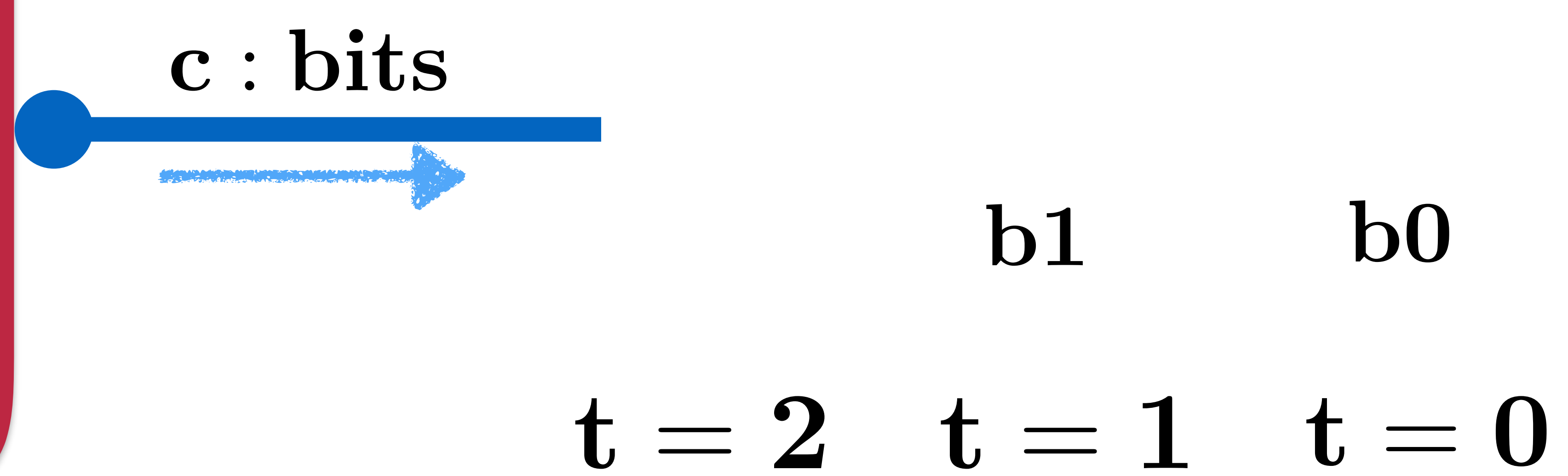

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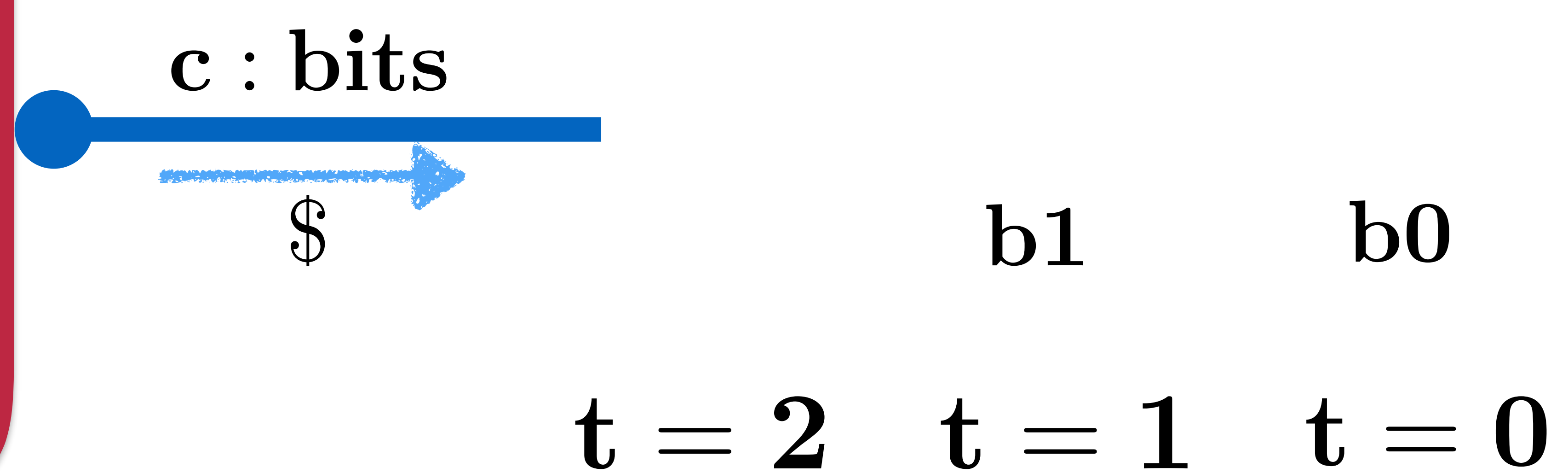

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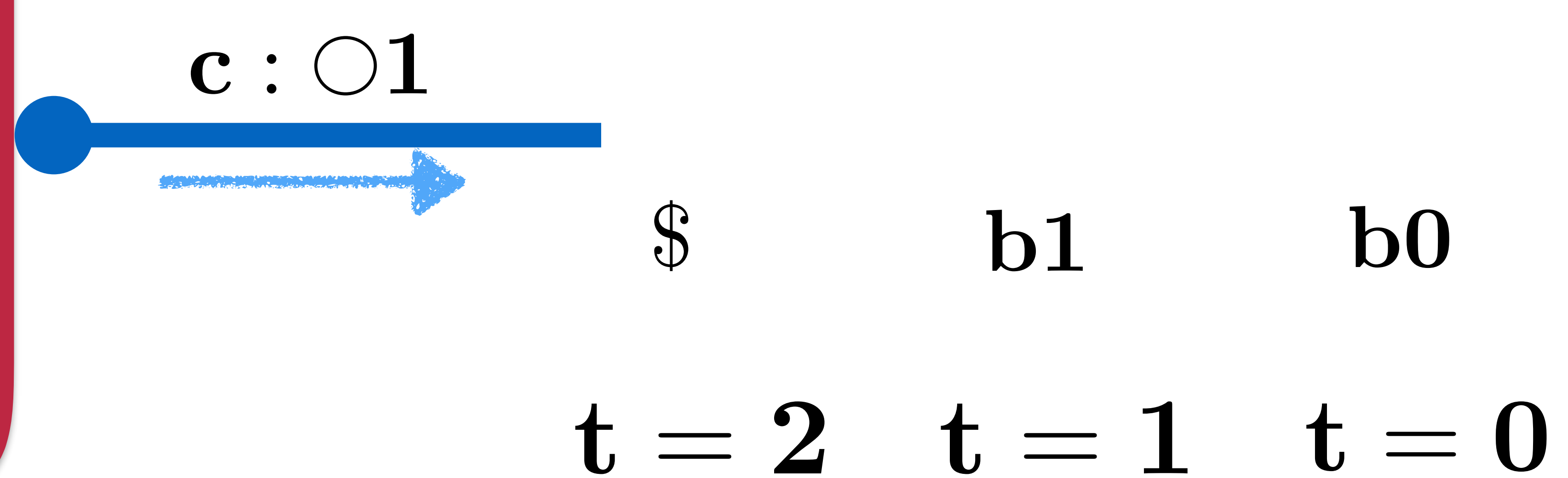

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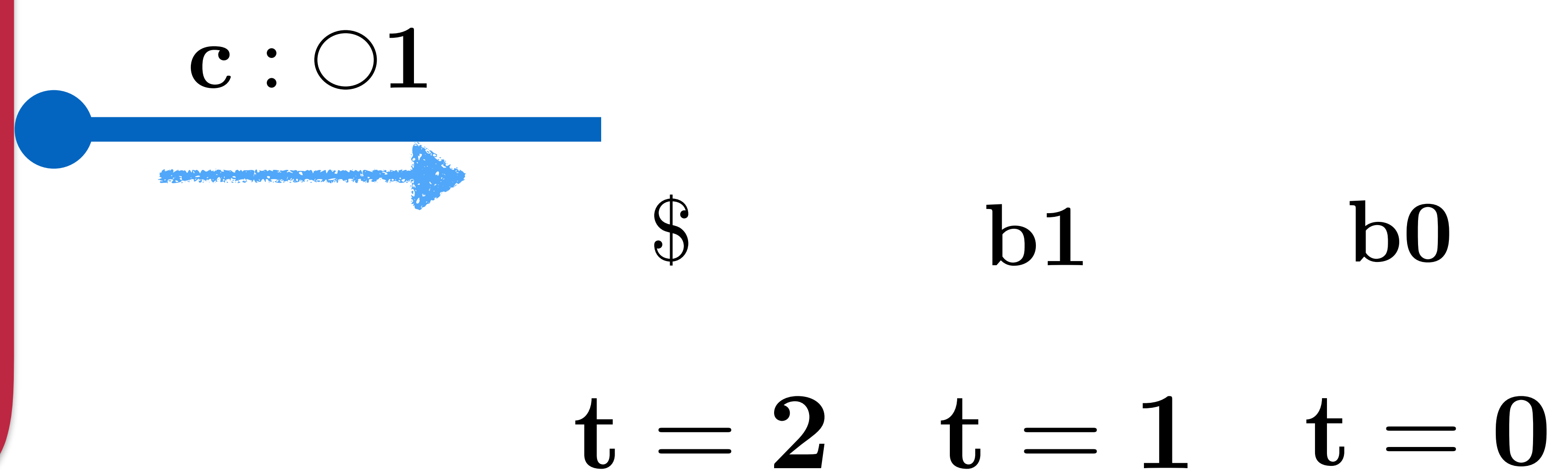

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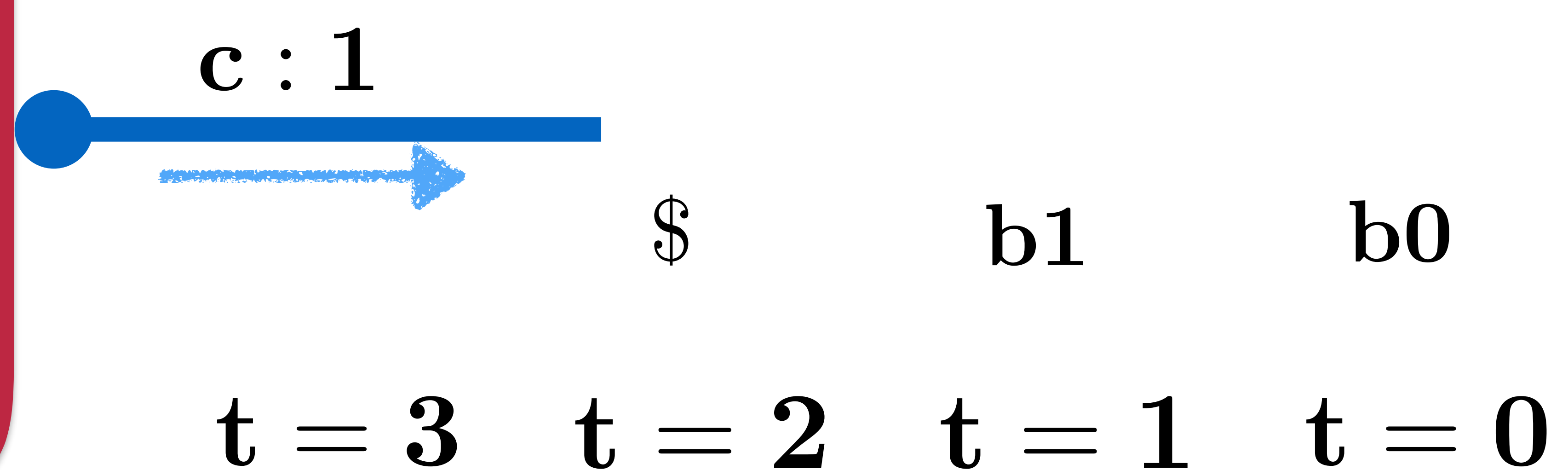

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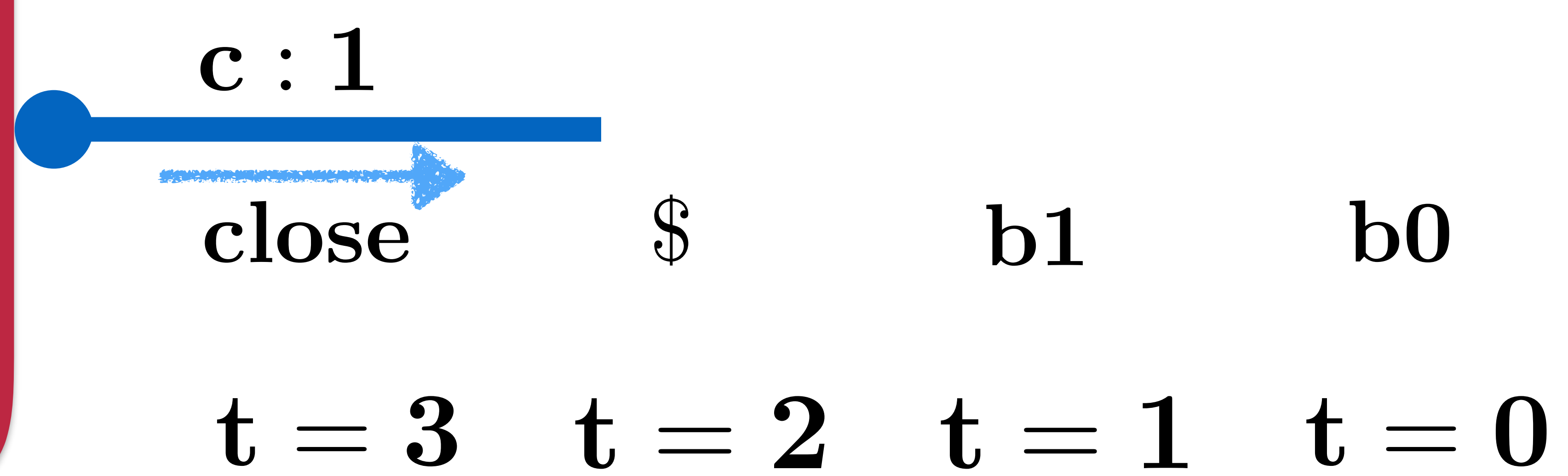

Typing Rule (○)

applied pointwise

$$\frac{\mathbf{\Omega} \vdash \mathbf{P} :: (\mathbf{x} : \mathbf{S})}{\bigcirc\mathbf{\Omega} \vdash \mathbf{delay};\ \mathbf{P} :: (\mathbf{x} : \bigcirc\mathbf{S})}$$

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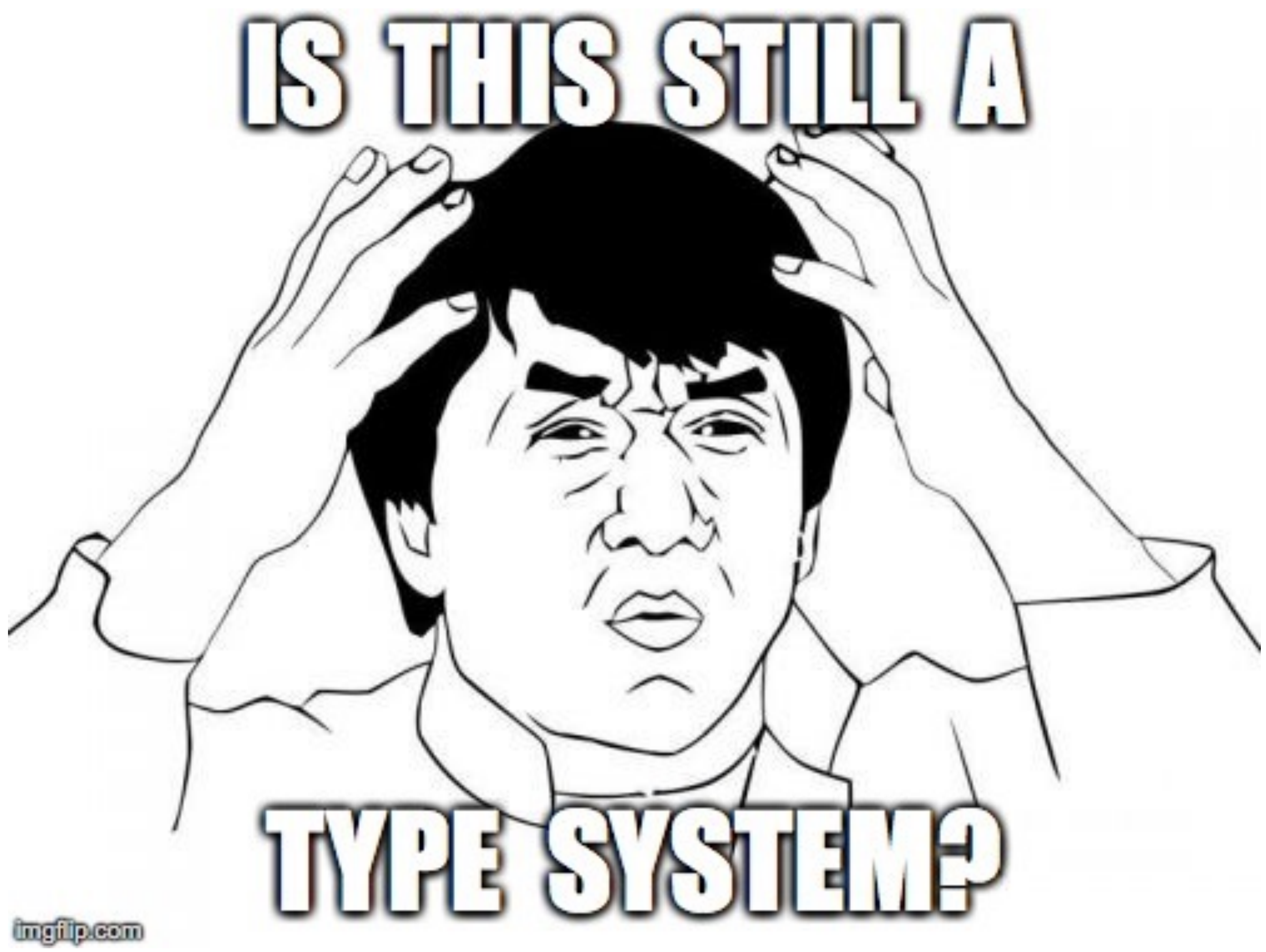
breaks the locality property of type system!



Bit Streams

$\mathcal{R}$ **cost model**

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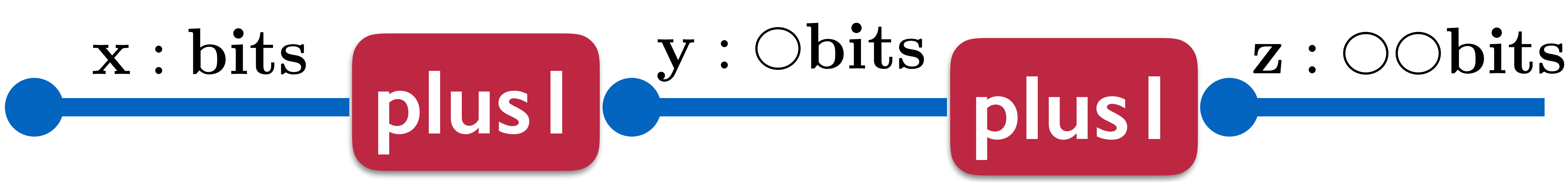
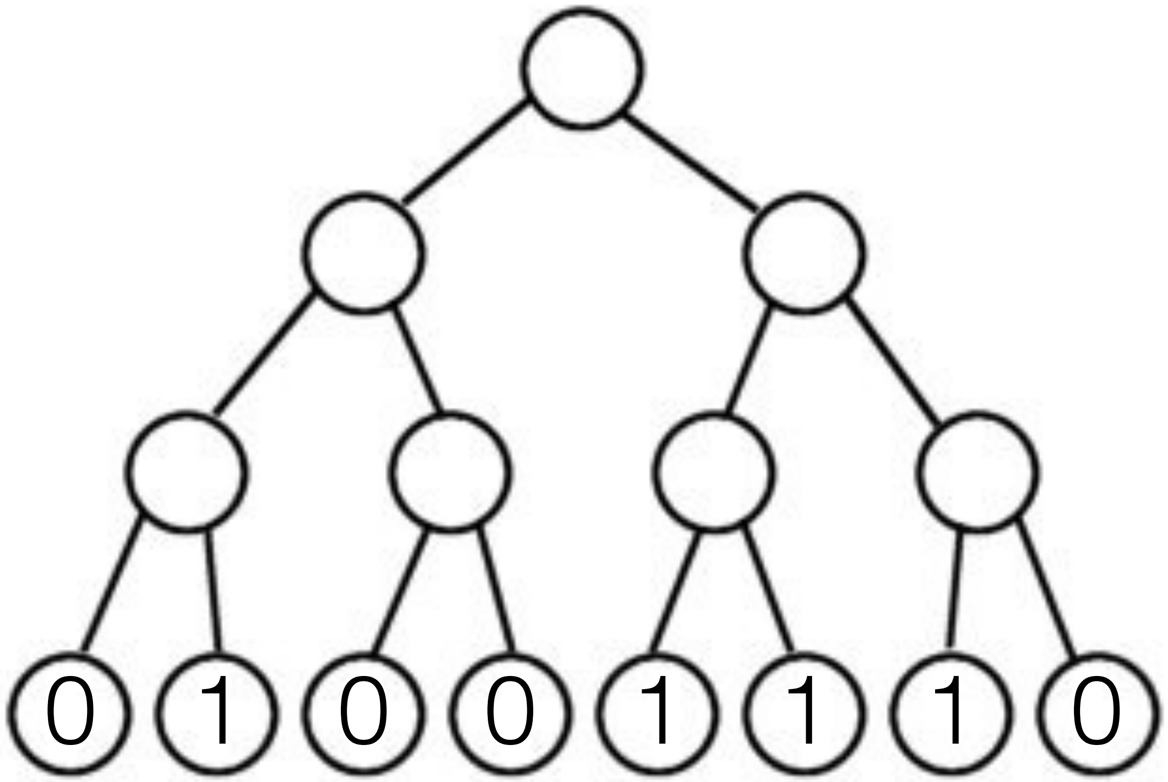


$\mathbf{x} : \mathbf{bits} \vdash \mathbf{plus2} :: (\mathbf{z} : \bigcirc\bigcirc\mathbf{bits})$

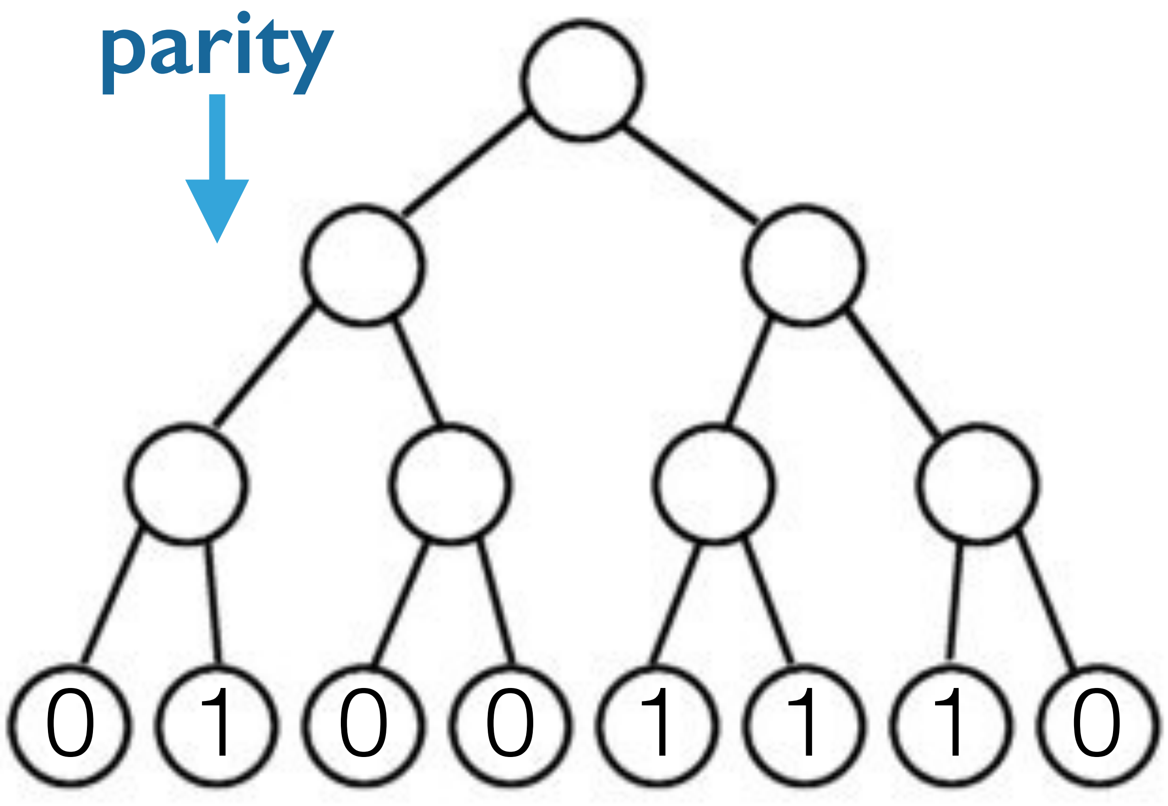
Fork-Join Parallelism



0s and 1s at the leaves

Compute the parity

Fork-Join Parallelism



0s and 1s at the leaves

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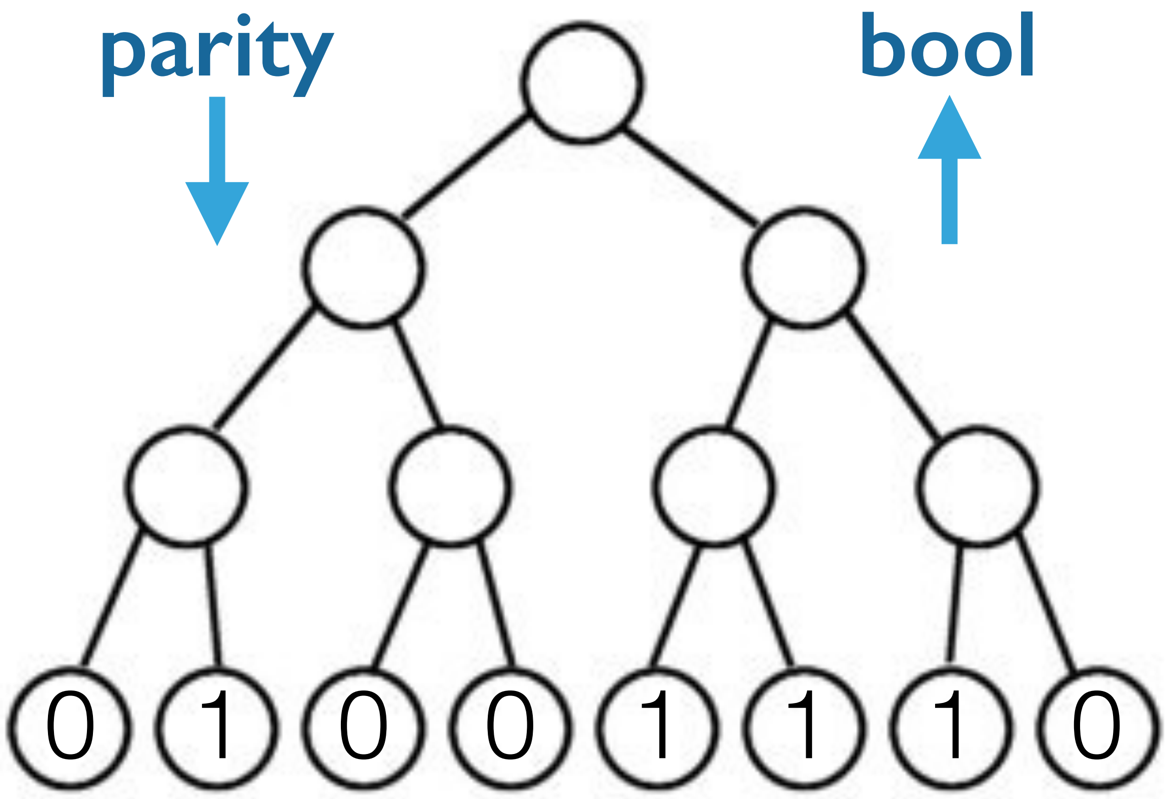

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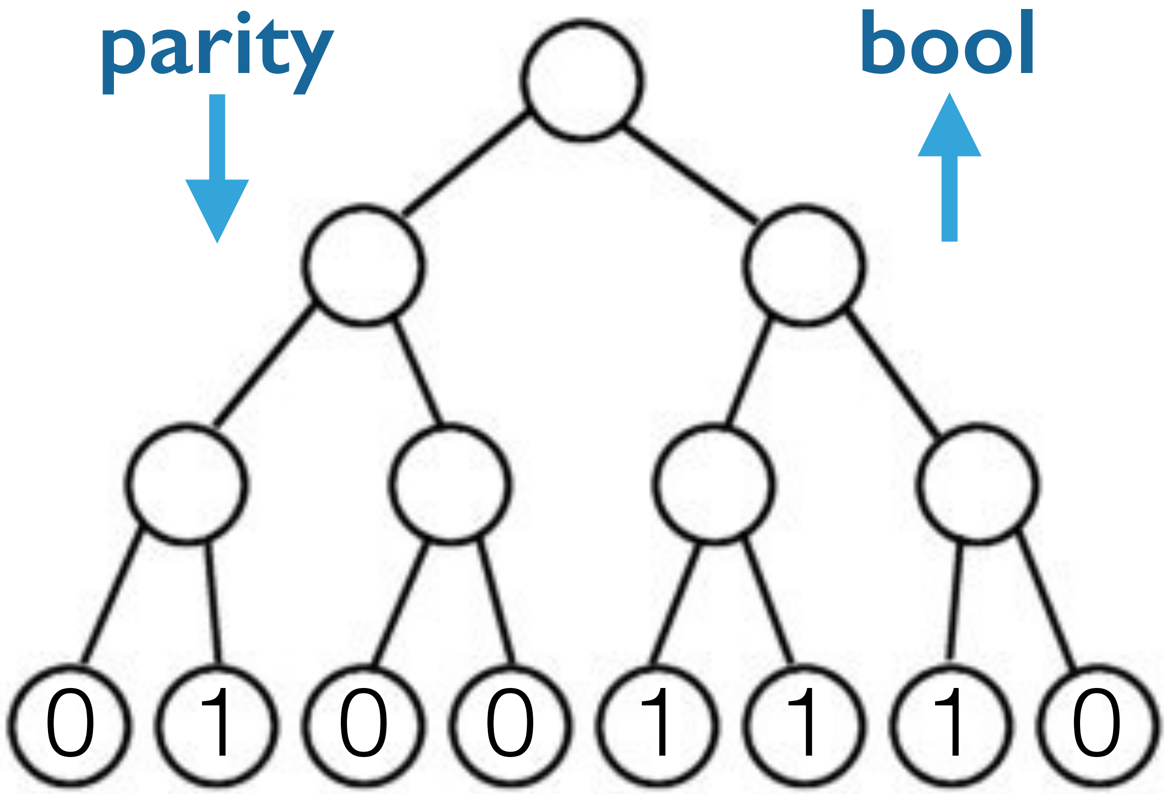

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$\mathcal{RS}$ cost model

$$\mathbf{tree[h]} = \&\{\mathbf{parity} : \bigcirc^{\mathbf{5h+3}}\mathbf{bool}\}$$

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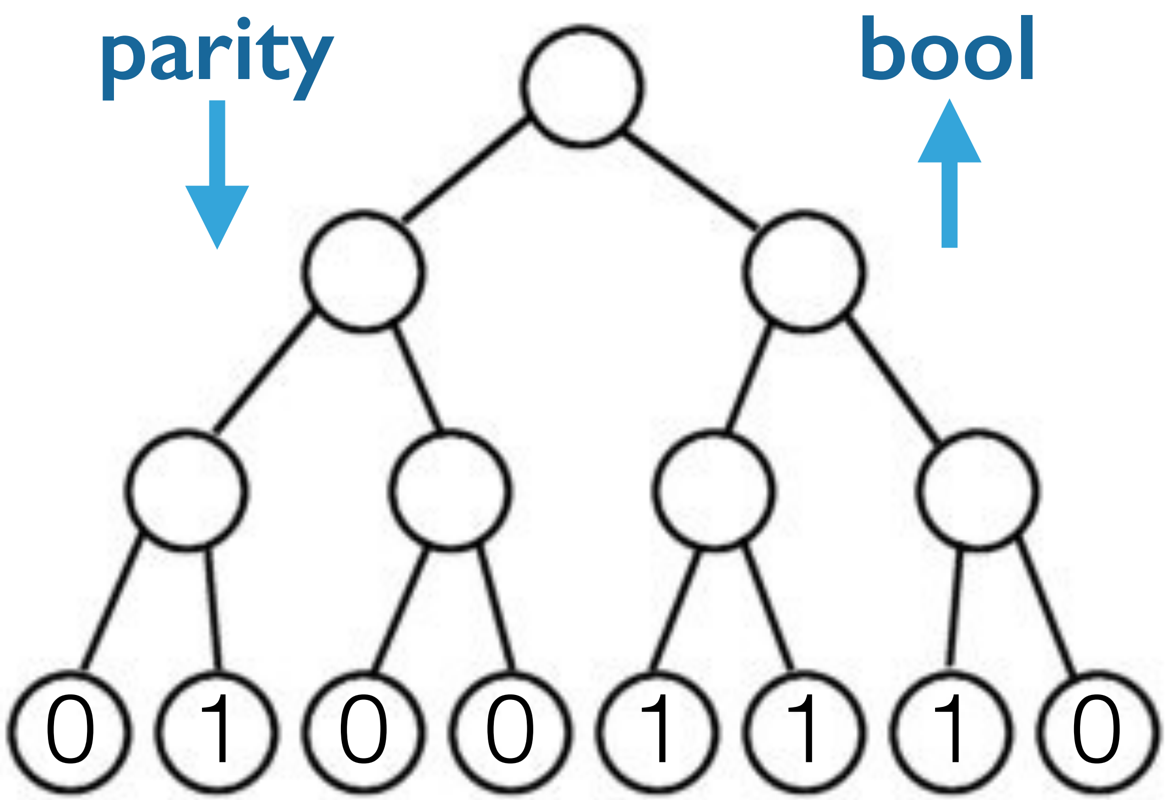

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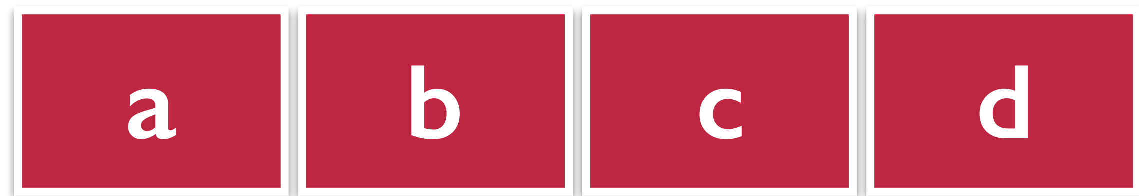
Counting xors $\quad \mathbf{tree[h]} = \&\{\mathbf{parity} : \bigcirc^{\mathbf{h}}\mathbf{bool}\}$

Can we type the queue?



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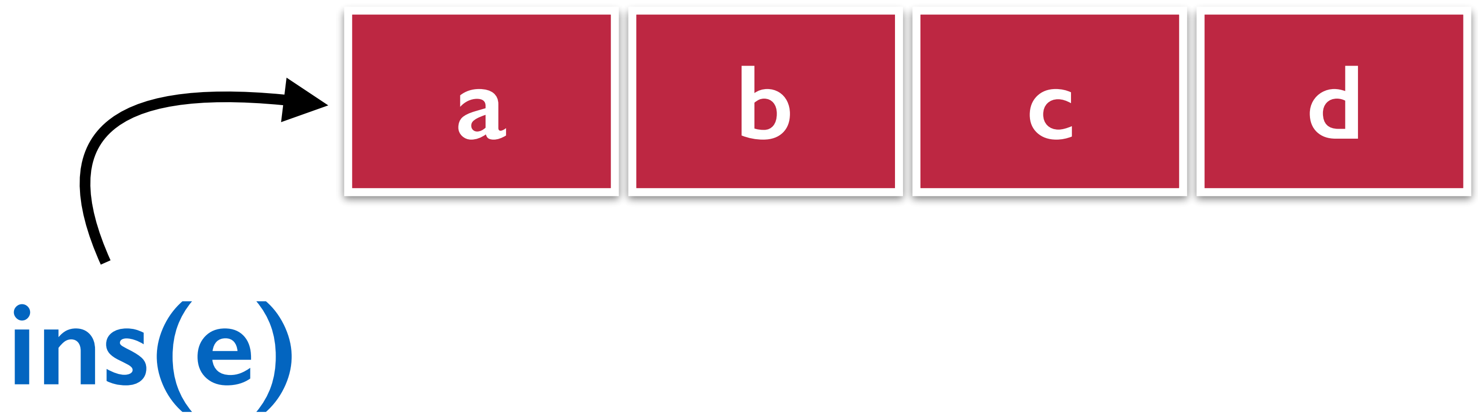

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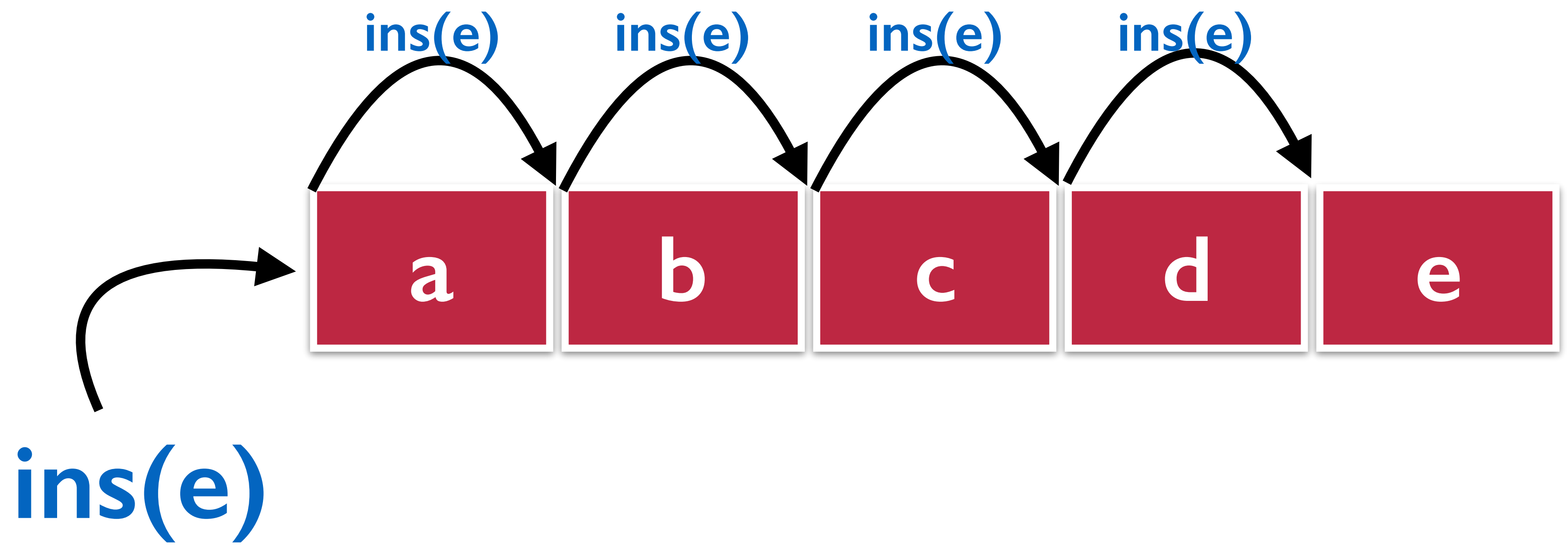

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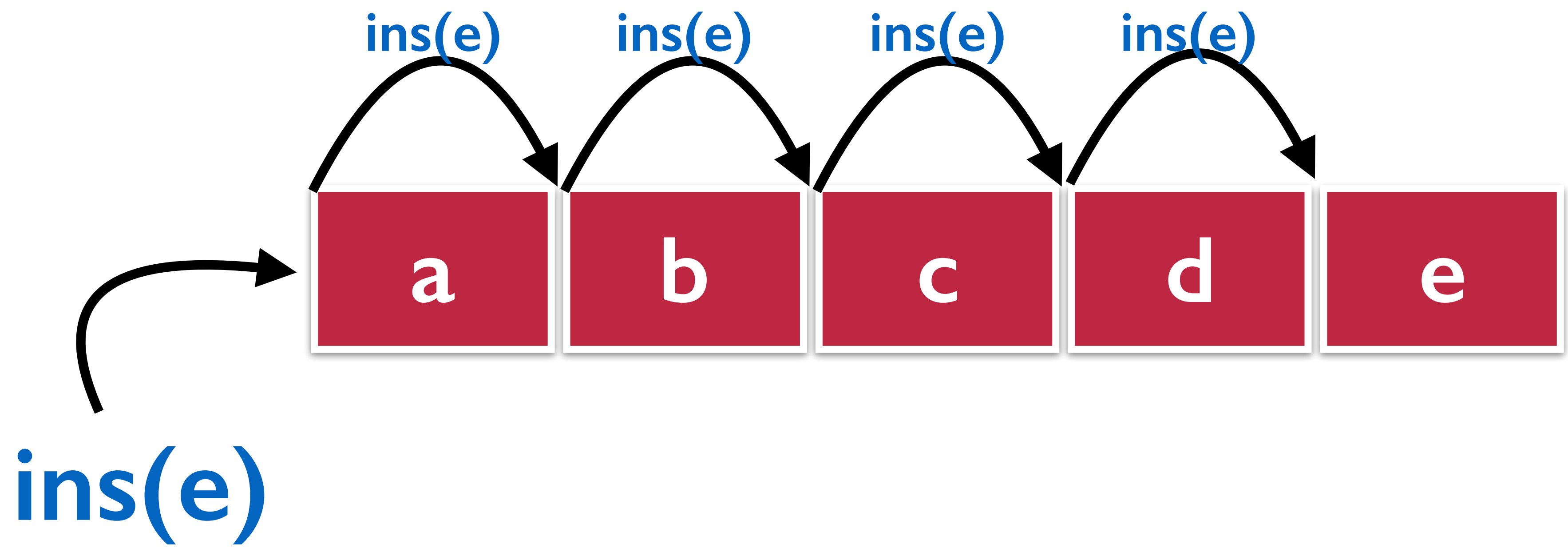

- Next operator only expresses constant insertion rate
- But rate of insertion at the tail depends on the size of the queue — longer the queue, slower the rate
- To maintain a constant rate at the tail, new elements must be inserted at a faster rate than the previous one

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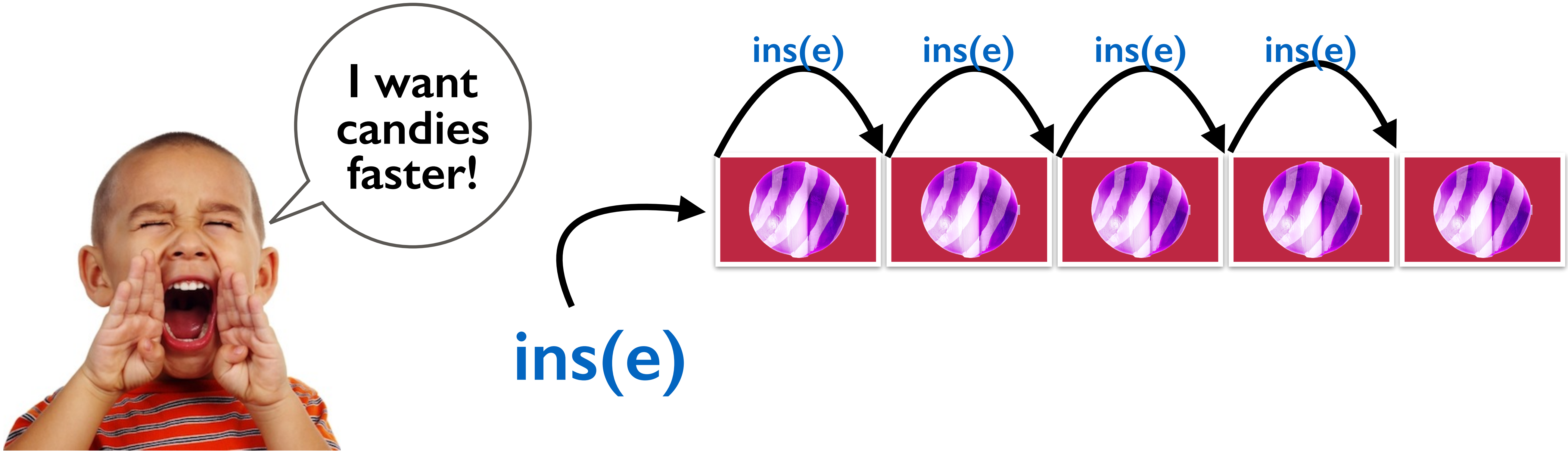

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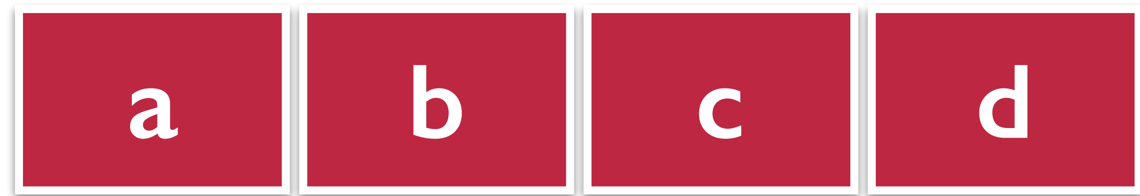
The Next Operator is too precise!

Adding Flexibility to the Type System

Providing Flexibility

- **The Box Operator (□)**
  - Provider Action: always be ready to receive token
  - Client Action: eventually send the token
  - Provider doesn't know when the token will come, only the client does
  - Different from ◯ operator where both provider and client knew the timing of message exchange
- **The Diamond Operator (◇)**
  - Dual of the Box operator (provider and client flip)

Typing the Queue



$$\mathsf{queue}_{\mathbf{A}} = \&\{\mathsf{ins} : \mathbf{A} \multimap \mathsf{queue}_{\mathbf{A}},\ \mathsf{del} : \oplus\{\mathsf{none} : \mathbf{1},\ \mathsf{some} : \mathbf{A} \otimes \mathsf{queue}_{\mathbf{A}}\}\}$$

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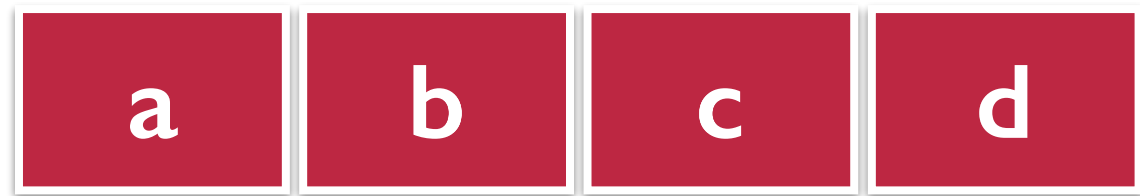

offers choice of ins/del

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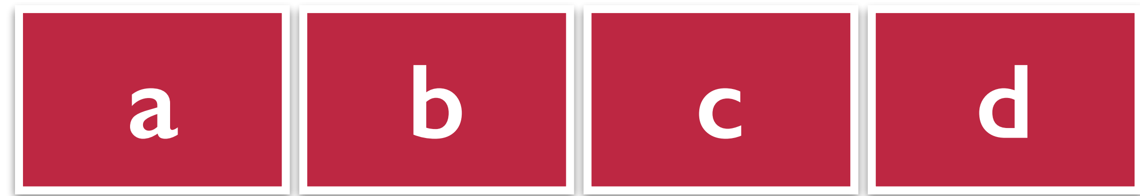

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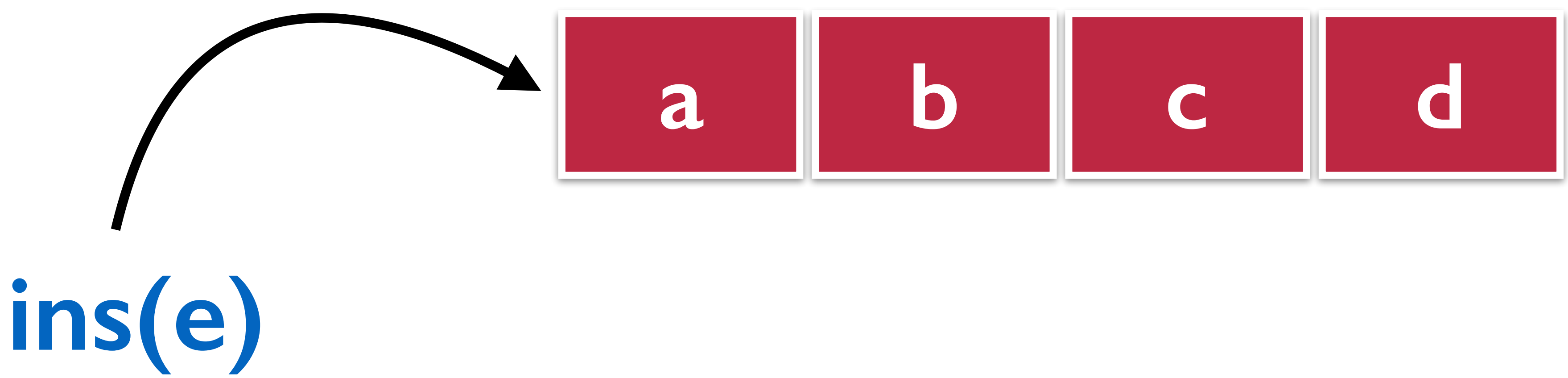

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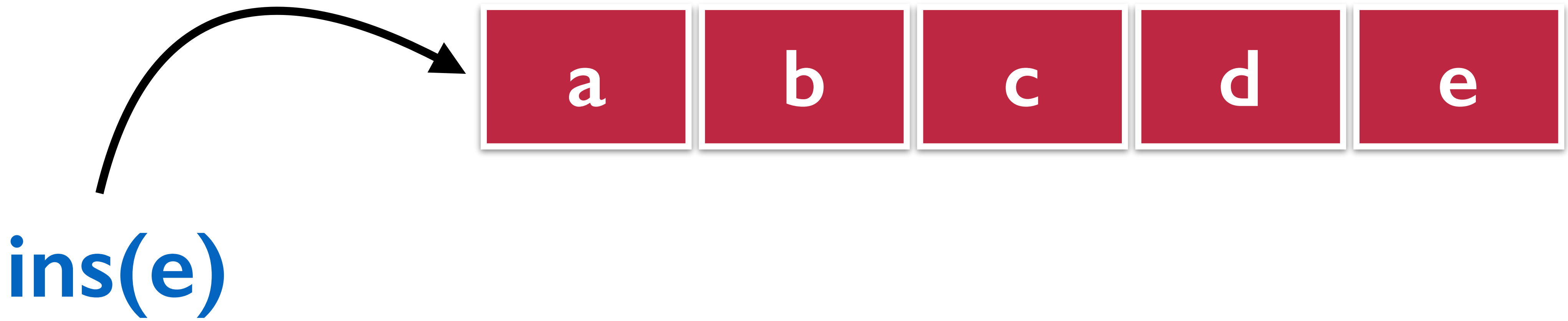

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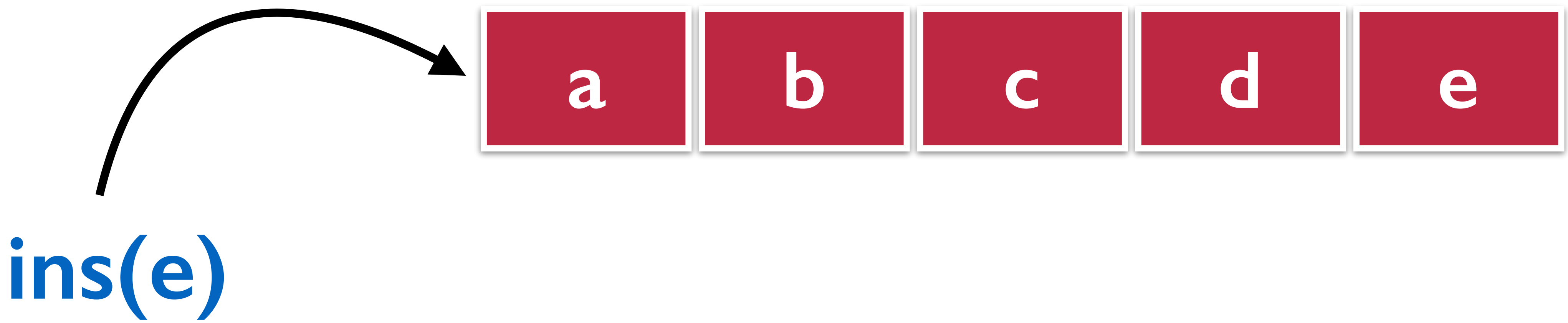

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behave as queue again

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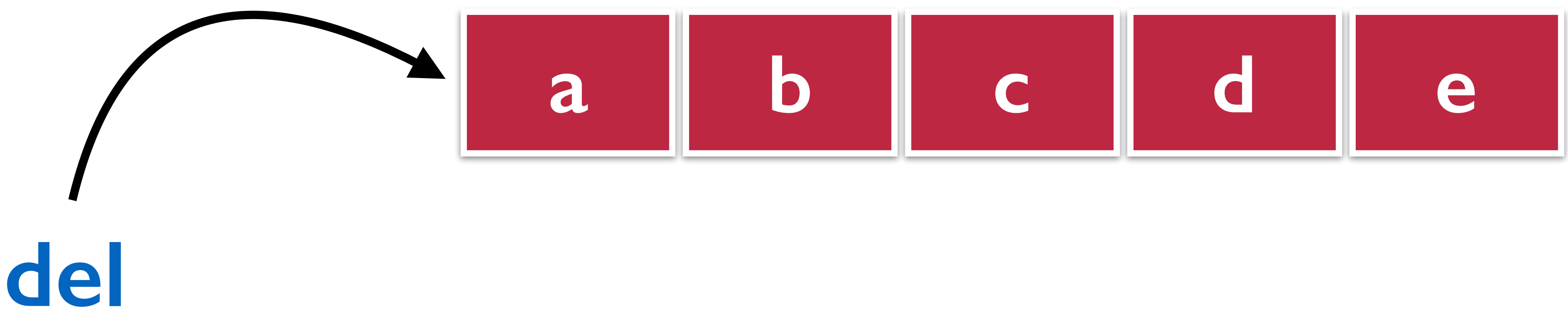

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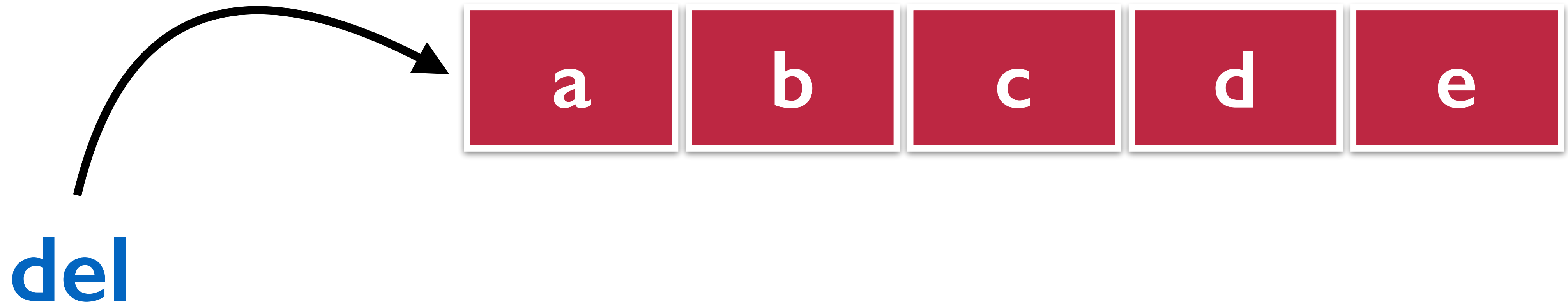

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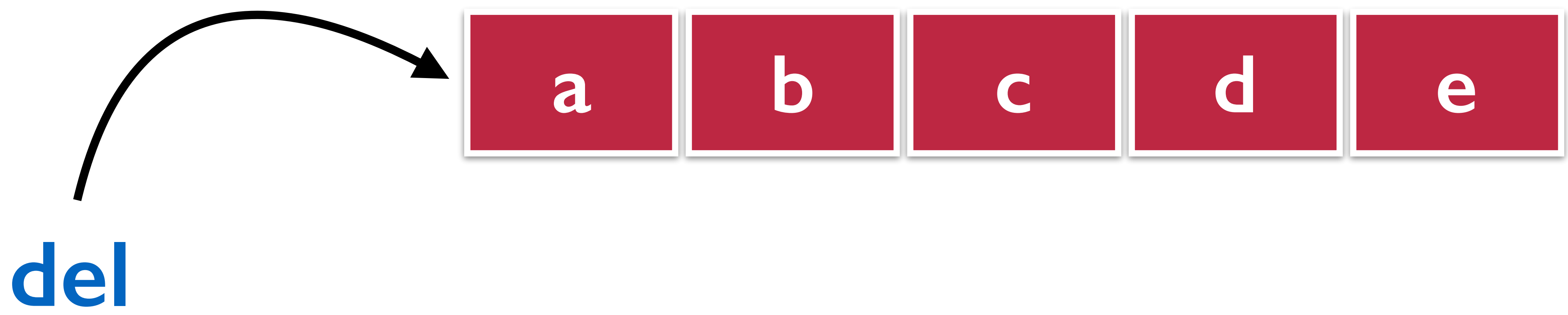
send none if queue is empty

Typing the Queue



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terminate

send none if queue is empty

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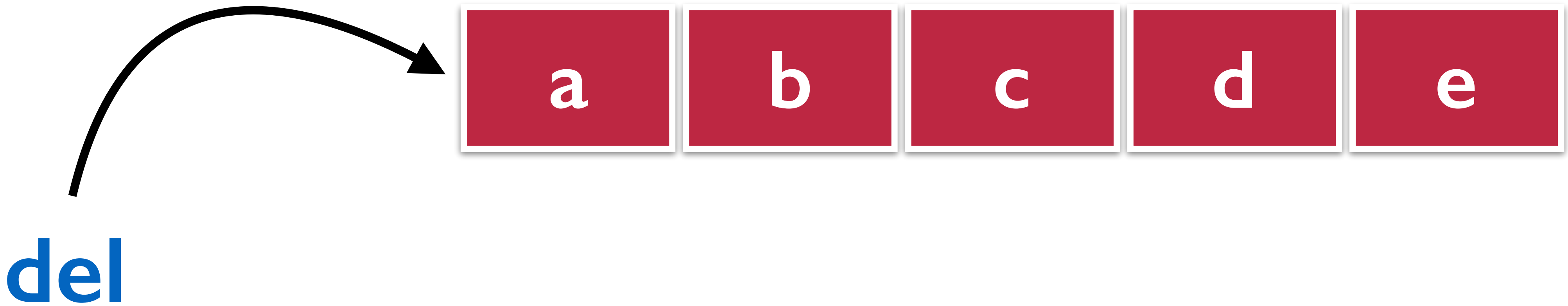

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send some otherwise

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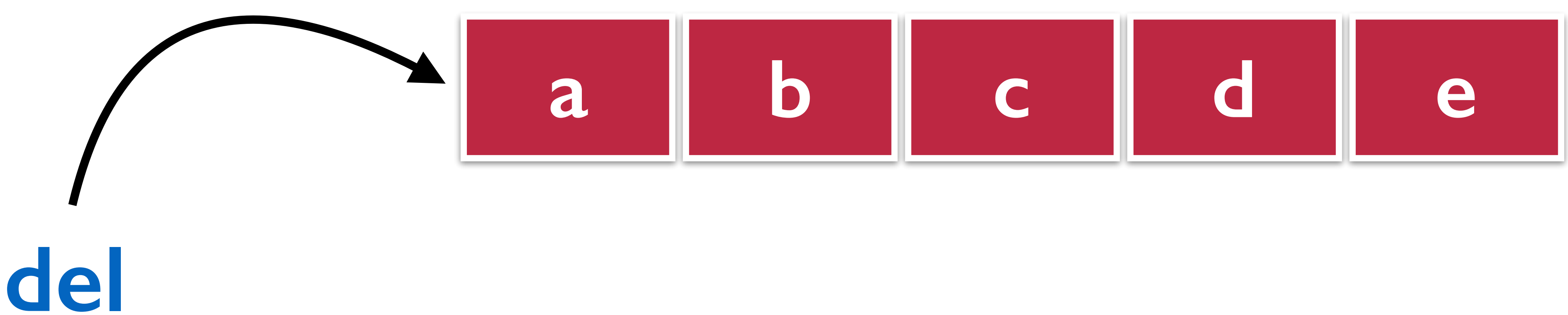

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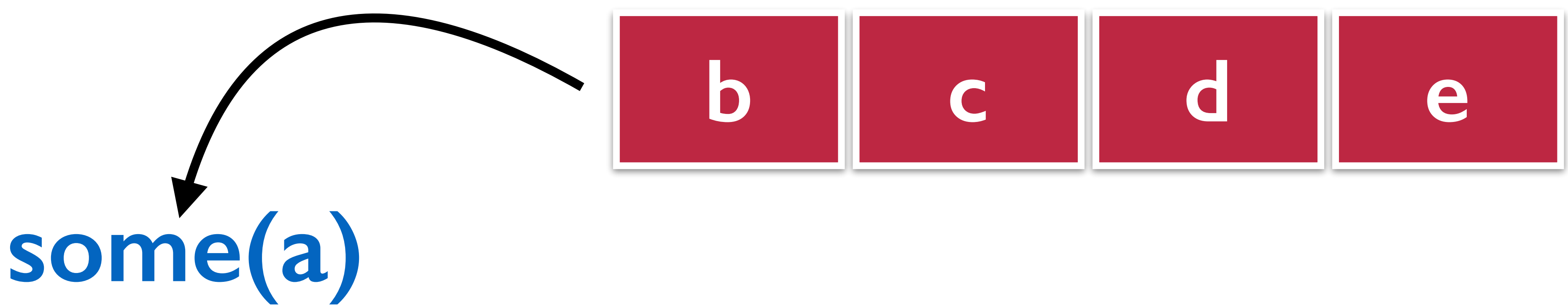

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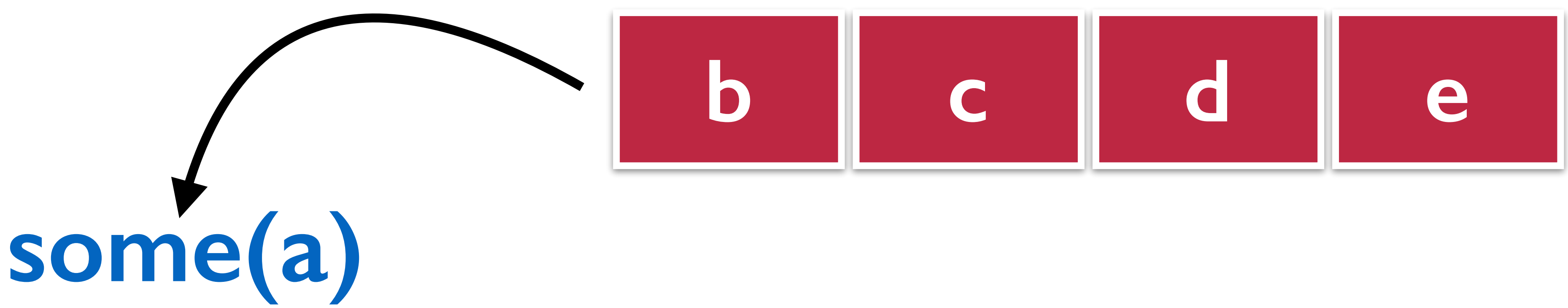

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Response Time of Queues

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Precision

WE ARE HERE!

Flexibility

Typing Rules(□)

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Exchanged token is a now! message

$$\frac{\Omega \text{ delayed}^{\Box} \quad \Omega \vdash P :: (x : S)}{\Omega \vdash \text{when? } x \,;\, P :: (x : \Box S)} \Box R$$

$\text{delayed}^{\Box} = \bigcirc^{*}\Box T$ → can be delayed indefinitely

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Stacks vs Queues

$\mathcal{RS}$ **cost model**

$$\begin{aligned}\mathbf{stack_A} = \Box\ \&\{&\mathbf{ins} : \bigcirc(\Box \mathbf{A} \multimap \bigcirc \mathbf{stack_A}),\\ &\mathbf{del} : \bigcirc \oplus \{\mathbf{none} : \bigcirc \mathbf{1},\\ &\qquad \mathbf{some} : \bigcirc(\Box \mathbf{A} \otimes \bigcirc \mathbf{stack_A})\}\}\end{aligned}$$

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Which one's more efficient?

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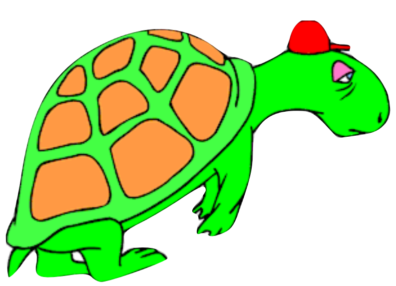

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Features of Type System

- **Parametric:** time can be defined using a cost model
- **Compositional:** types describe individual processes, not just whole programs
- **Precise & Flexible:** $\bigcirc$ operator provides precision. $\Box$, $\Diamond$ operators provide flexibility
- **Conservative:** only added 3 type operators
- **General:** works on all standard examples
- **Automatic:** supports automatic type checking, type inference future work

Cost Semantics

$$\mathbf{proc}(\mathbf{c}, \mathbf{t}, \mathbf{P})$$

Process P offering along channel c at local time t

Cost Semantics

$$\mathrm{proc}(c, t, P)$$

Process P offering along channel c at local time t

Soundness Theorem:

message timings realized by the local clocks matches the timing predicted by the type system

What else is in the paper?

- Interaction of $\Box$, $\Diamond$ with $\bigcirc$ operators
- Sound and complete *subtyping* relation
- *Time Reconstruction* — inserting delay, now!, when? automatically from the program type
- *Cost Semantics* — each process stores a *local clock*, expresses timing at runtime, connected to the type system by a proof of *progress* and *preservation*
- Connection to the standard cost semantics
- Typing a set of processes at *different* local clocks

Conclusion

Conclusion

Type System

analyzes timing of message exchanges

Soundness Theorem

Cost Semantics

local clocks at each process

Properties

conservative extension, added 3 type operators

$\bigcirc$ provides precision, $\Box$, $\Diamond$ provide flexibility

Examples

throughput and latency of bit stream processors

response time of stacks vs queues

list examples: append, map, fold (many more in paper!)