CS 599 A1: Assignment 4

Due: Thursday, April 2, 2026

Total: 100 pts

Instructor: Ankush Das

- This assignment is due midnight on the above date and must be submitted electronically on Gradescope.
- You are provided a tex file, named asgn4.tex. It contains environments called solution to enter your solutions. And please include your name and BU ID in the author section (above).
- Although not recommended, you can submit handwritten answers scanned as a PDF and clearly legible.

1 Contraction-Free Sequent Calculus

Problem 1 (Mandatory; No LLMs; 20 pts) *Prove the soundness of contraction-free sequent calculus, i.e., if* $\Gamma \longrightarrow A$*, then* $\Gamma \Longrightarrow A$*.*

Problem 2 (Mandatory; No LLMs; 30 pts) *Complete the proof of completeness of contraction-free sequent calculus, i.e., i.e., if* $\Gamma \Longrightarrow A$*, then* $\Gamma \longrightarrow A$*. You only need to show the case when the derivation of* $\Gamma \Longrightarrow A$ *ends in* $\supset L$ *rule.*

2 Contraction-Free Sequent Calculus with Inversion

In this problem, you need to design a new calculus that combines the inversion calculus with contraction-free sequent calculus. We define that the inversion property holds for a rule if we can apply the rule in bottom-up proof construction without backtracking whenever it contains the principal formula in the conclusion of the rule. For most rules, it should be clear what the principal formula is (for example, $A \vee B$ in $\vee R_1$ and $(A_1 \supset A_2) \supset B$ in $\supset\supset L$). For id^*, we consider the succedent principal, and for $P \supset L$, it is $P \supset B$.

The contraction-free sequent calculus with inversion should satisfy the following properties:

1. It should be immediately sound with respect to the contraction-free sequent calculus in the sense that it has analogous rules, plus some structural rules.
2. When inversion applies as defined above, exactly one rule should be applicable.
3. When inversion does not apply, it should offer a complete set of choices among the remaining rules. By "complete" we mean that if a sequent is provable in the contraction-free sequent calculus, it should be provable in your calculus.

Problem 3 (Mandatory; No LLMs; 50 pts) *Define your judgments and write out* ***all the rules****.*

Hint: *It would help to define at least three judgments: right inversion, left inversion, and choice.*

3 Optional Problems

Problem 4 (Optional; LLMs ok; 20 pts) *Prove identity expansion for the inversion calculus. Concretely, show that* $\epsilon\,;\,A \xrightarrow{\mathsf{R}} A$ *holds for any arbitrary proposition* A *in the inversion calculus.*

Problem 5 (Optional; LLMs ok; 10 pts) *Does* $\epsilon\,;A \xrightarrow{\mathsf{L}} A$ *hold for any arbitrary proposition* A *in the inversion calculus? Briefly justify your answer. If it does not hold, give a counterexample* A *for which this statement does not hold.*

Problem 6 (Optional; LLMs ok; 10 pts) *Does* $\epsilon\,;A \xrightarrow{\mathsf{C}} A$ *hold for any arbitrary proposition* A *in the inversion calculus? Briefly justify your answer. If it does not hold, give a counterexample* A *for which this statement does not hold.*