CS 599 A1: Assignment 3

Due: Monday, March 2, 2026

Total: 100 pts

Instructor: Ankush Das

- This assignment is due midnight on the above date and must be submitted electronically on Gradescope.
- You are provided a tex file, named asgn3.tex. It contains environments called solution to enter your solutions. And please include your name and BU ID in the author section (above).
- Although not recommended, you can submit handwritten answers scanned as a PDF and clearly legible.
- There are no optional questions in this assignment and LLMs are not permitted for any questions.

Definition. $\neg A$ is defined as $A \supset \bot$.

Problem 1 (Mandatory; No LLM; 60 pts) *For the following sequents, either provide a proof using rules of sequent calculus or explain why none exists.*

(i) $(A \supset (B \supset C)) \Longrightarrow (A \wedge B) \supset C$

(ii) $A \Longrightarrow \neg\neg A$

(iii) $(\neg A \vee \neg B) \Longrightarrow \neg(A \wedge B)$

(iv) $(A \wedge (B \vee C)) \Longrightarrow (A \wedge B) \vee (A \wedge C)$

(v) $A \supset B \Longrightarrow \neg B \supset \neg A$

(vi) $(A \supset B) \supset B \Longrightarrow \neg\neg A \supset B$

Problem 2 (Mandatory; No LLM; 20 pts) *Provide sequent calculus rules for universal and existential quantification. In particular, provide:*

(i) Right rule for $\Gamma \Longrightarrow \forall x.A(x)$.

(ii) Left rule for $\Gamma, \forall x.A(x) \Longrightarrow C$.

(iii) Right rule for $\Gamma \Longrightarrow \exists x.A(x)$.

(iv) Left rule for $\Gamma, \exists x.A(x) \Longrightarrow C$.

Problem 3 (Mandatory; No LLMs; 20 pts) *Recall the natural deduction rules for Heyting artithmetic*

$$\frac{}{\mathsf{zero} : \mathsf{nat}}\ \textsc{NatI}_z \qquad \frac{n : \mathsf{nat}}{\mathsf{succ}\ n : \mathsf{nat}}\ \textsc{NatI}_s \qquad \frac{n : \mathsf{nat} \quad C(0)\ \mathsf{true} \quad \begin{array}{c} \overline{x : \mathsf{nat}} \quad \overline{C(x)\ \mathsf{true}}^{\,y} \\ \vdots \\ C(\mathsf{succ}\ x)\ \mathsf{true} \end{array}}{C(n)\ \mathsf{true}}\ \textsc{NatE}^{x,y}$$

The elimination rule closely resembles the principle of mathematical induction. But natural numbers are just one example of inductive data structures. In this problem, we consider two different inductive data structures, lists and trees. For your convenience, the introduction rules are provided.

- *The introduction rules for lists are defined as follows:*

$$\frac{}{\textsf{nil} : \tau\ \textsf{list}}\ \textsc{ListI}_n \qquad\qquad \frac{x : \tau \qquad \ell : \tau\ \textsf{list}}{\textsf{cons}(x, \ell) : \tau\ \textsf{list}}\ \textsc{ListI}_c$$

  Design the corresponding elimination rule for lists.

- *The introduction rules for trees are defined as follows:*

$$\frac{}{\textsf{leaf} : \tau\ \textsf{tree}}\ \textsc{TreeI}_l \qquad\qquad \frac{x : \tau \qquad \ell : \tau\ \textsf{tree} \qquad r : \tau\ \textsf{tree}}{\textsf{node}(x, \ell, r) : \tau\ \textsf{tree}}\ \textsc{TreeI}_n$$

  Design the corresponding elimination rule for trees.

Hint: how would induction work on lists and trees?