CS 599 A1: Assignment 2

Due: Thursday, February 19, 2026

Total: 100 pts

Instructor: Ankush Das

- This assignment is due midnight on the above date and must be submitted electronically on Gradescope.
- You are provided a `tex` file, named `asgn2.tex`. It contains environments called `solution` to enter your solutions. And please include your name and BU ID in the author section (above).
- Although not recommended, you can submit handwritten answers scanned as a PDF and clearly legible.
- Each problem specifies whether LLMs are allowed for it or not. Please adhere to it.
- Each problem is marked whether it is mandatory or optional. You must solve some of the optional problems to get to 100 points.

Definition. $\neg A$ is defined as $A \supset \bot$.

Problem 1 (Mandatory; No LLM; 10 pts) *Recall the natural deduction problems from Assignment 1. For the following propositions, provide the corresponding proof term* [2.5 pts each].

(i) $(A \supset (B \supset C)) \supset (A \wedge B) \supset C$

(ii) $A \supset \neg\neg A$

(iii) $(\neg A \vee \neg B) \supset \neg(A \wedge B)$

(iv) $(A \wedge (B \vee C)) \supset (A \wedge B) \vee (A \wedge C)$

Problem 2 (Mandatory; No LLM; 15 pts) *Recall the $A \wedge B$ operator from constructive logic. Suppose we keep the introduction rule unchanged, but update the elimination rule as follows:*

$$\dfrac{A\ \mathsf{true} \qquad B\ \mathsf{true}}{A \wedge B\ \mathsf{true}}\ \wedge I \qquad\qquad \dfrac{A \wedge B\ \mathsf{true} \qquad \begin{array}{c} \overline{A\ \mathsf{true}}^{\,x} \quad \overline{B\ \mathsf{true}}^{\,y} \\ \vdots \\ C\ \mathsf{true} \end{array}}{C\ \mathsf{true}}\ \wedge E^{x,y}$$

- *Are the rules locally sound? If yes, show a reduction. Otherwise, give an explanation of unsoundness.*
- *Are the rules locally complete? If yes, show an expansion. Otherwise, give an explanation of incompleteness.*
- *Provide an appropriate proof term for the elimination rule.*

Problem 3 (Mandatory; No LLM; 15 pts) *Recall the $A \supset B$ operator from constructive logic. Suppose we keep the introduction rule unchanged, but update the elimination rule as follows:*

$$\dfrac{\begin{array}{c} \overline{A\ \mathsf{true}}^{\,x} \\ \vdots \\ B\ \mathsf{true} \end{array}}{A \supset B\ \mathsf{true}}\ \supset I^x \qquad\qquad \dfrac{A \supset B\ \mathsf{true} \qquad A\ \mathsf{true} \qquad \begin{array}{c} \overline{B\ \mathsf{true}}^{\,x} \\ \vdots \\ C\ \mathsf{true} \end{array}}{C\ \mathsf{true}}\ \supset E^x$$

- *Are the rules locally sound? If yes, show a reduction. Otherwise, give an explanation of unsoundness.*
- *Are the rules locally complete? If yes, show an expansion. Otherwise, give an explanation of incompleteness.*
- *Provide an appropriate proof term for the elimination rule.*

Problem 4 (Mandatory; No LLMs; 15 pts) *Suppose we define a new operator $A \star B$ with the following introduction and elimination rules:*

$$\dfrac{\begin{array}{c}\overline{A\ \mathsf{true}}^{\,x}\\ \vdots\\ B\ \mathsf{true}\end{array}}{A \star B\ \mathsf{true}}\ \star I_1^x \qquad\qquad \dfrac{\begin{array}{c}\overline{B\ \mathsf{true}}^{\,x}\\ \vdots\\ A\ \mathsf{true}\end{array}}{A \star B\ \mathsf{true}}\ \star I_2^x$$

$$\dfrac{A \star B\ \mathsf{true} \qquad A\ \mathsf{true} \qquad \begin{array}{c}\overline{B\ \mathsf{true}}^{\,x}\\ \vdots\\ C\ \mathsf{true}\end{array} \qquad B\ \mathsf{true} \qquad \begin{array}{c}\overline{A\ \mathsf{true}}^{\,y}\\ \vdots\\ C\ \mathsf{true}\end{array}}{C\ \mathsf{true}}\ \star E^{x,y}$$

- *Are the rules locally sound? If yes, show a reduction. Otherwise, give an explanation of unsoundness.*
- *Are the rules locally complete? If yes, show an expansion. Otherwise, give an explanation of incompleteness.*
- *Can $A \star B$ be expressed using the standard operators in constructive logic?*

Problem 5 (Mandatory; No LLMs; 15 pts) *Suppose we define a new operator $A \heartsuit B \Diamond C$ with the following introduction and elimination rules:*

$$\dfrac{A\ \mathsf{true}}{A \heartsuit B \Diamond C\ \mathsf{true}}\ \heartsuit\Diamond I_1 \qquad\qquad \dfrac{B\ \mathsf{true} \qquad C\ \mathsf{true}}{A \heartsuit B \Diamond C\ \mathsf{true}}\ \heartsuit\Diamond I_2$$

$$\dfrac{A \heartsuit B \Diamond C\ \mathsf{true} \qquad \begin{array}{c}\overline{B\ \mathsf{true}}^{\,x} \quad \overline{C\ \mathsf{true}}^{\,y}\\ \vdots\\ D\ \mathsf{true}\end{array} \qquad \begin{array}{c}\overline{A\ \mathsf{true}}^{\,z}\\ \vdots\\ D\ \mathsf{true}\end{array}}{D\ \mathsf{true}}\ \heartsuit\Diamond E^{x,y,z}$$

- *Are the rules locally sound? If yes, show a reduction. Otherwise, give an explanation of unsoundness.*
- *Are the rules locally complete? If yes, show an expansion. Otherwise, give an explanation of incompleteness.*
- *Provide a derivation of $A \heartsuit B \Diamond C \supset A \heartsuit C \Diamond B$ true.*

Problem 6 (Optional; No LLMs; 15 pts) *Determine if the following propositions are derivable using rules of verifications ($A\uparrow$) and uses ($A\downarrow$). If yes, provide a derivation. Otherwise, briefly explain why.*

(i) $((A \vee B) \wedge (A \supset C) \wedge (B \supset C)) \supset C \uparrow$

(ii) $(A \supset B) \supset ((A \vee C) \supset (B \vee C)) \uparrow$

(iii) $(A \vee B) \supset ((A \supset C) \supset ((B \supset D) \supset (C \vee D))) \uparrow$

Problem 7 (Optional; LLMs ok; 30 pts) *Given a derivation $\mathcal{D}$ of A true, can you construct a derivation $\mathcal{E}$ of $A \uparrow$.*
Hint: Try this problem with only the $A \wedge B$ and $A \supset B$ operators first.

Problem 8 (Optional; LLMs ok; 15 pts) *Design introduction and elimination rules of a new connective $A \bullet B$ such that it is locally sound but not locally complete.*

Problem 9 (Optional; LLMs ok; 15 pts) *Design introduction and elimination rules of a new connective $A \circ B$ such that it is locally complete but not locally sound.*