CS 599 A1: Assignment 1

Due: Thursday, February 5, 2026

Total: 100 pts

Instructor: Ankush Das

- This assignment is due midnight on the above date and it must be submitted electronically on Gradescope. Please create an account on Gradescope, if you haven't already done so.
- Please use the template provided on the course webpage to typeset your assignment and please include your name and BU ID in the Author section (above).
- Although it is not recommended, you can submit handwritten answers that are scanned as a PDF and clearly legible.
- You are provided a `tex` file, named `asgn1.tex`. It contains environments called `solution`. Please enter your solutions inside these environments.
- Section 1 contains simpler exam-style questions that you should try to solve without using LLMs.
- Sections 2, 3, and 4 contain more open-ended questions for which you should feel free to use LLMs. Please acknowledge how you used LLMs in your answer.

1 Natural Deduction [40 pts]

Definition. $\neg A$ is defined as $A \supset \bot$.

Problem 1 (40 pts) *Determine whether each of the following propositions is derivable or not in constructive logic. If it is derivable, then provide a derivation using natural deduction. If it is not derivable, provide a brief explanation why* [5 pts each].

(i) $(A \supset (B \supset C)) \supset (A \wedge B) \supset C$

(ii) $(A \supset (B \vee C)) \supset (A \supset B) \vee (A \supset C)$

(iii) $(A \supset B) \vee (B \supset A)$

(iv) $A \supset \neg\neg A$

(v) $\neg\neg A \supset A$

(vi) $\neg(A \wedge B) \supset (\neg A \vee \neg B)$

(vii) $(\neg A \vee \neg B) \supset \neg(A \wedge B)$

(viii) $(A \wedge (B \vee C)) \supset (A \wedge B) \vee (A \wedge C)$

2 Equality with Naturals and Reals [20 pts]

Definition. Natural numbers, denoted by $\mathbb{N}$, is the set of numbers defined inductively as follows:

- 0 is a natural number, i.e., $0 \in \mathbb{N}$.
- If n is a natural number, so is $n+1$, i.e., $n \in \mathbb{N} \supset n+1 \in \mathbb{N}$.

In other words, natural numbers are "constructed" by successively adding 1 to 0.

Problem 2 (10 pts) *Given this definition, is the following proposition derivable in constructive logic? Justify your answer.*

$$\forall n \in \mathbb{N}.\ n = 0 \vee n \neq 0$$

(Note that this is trivially true in classical logic since it is of the form $A \vee \neg A$)

Problem 3 (10 pts) *What if we change the domain to real numbers $\mathbb{R}$? Is the following proposition derivable in constructive logic? Provide a brief and informal justification.*

$$\forall x \in \mathbb{R}.\ x = 0 \vee x \neq 0$$

3 More Fun with Natural Numbers [20 pts]

Definition. For natural numbers a and b, we define that a divides b, written as $a \mid b$, if there exists a natural number k such that $b = a \times k$. We define $a \nmid b$ as $\neg(a \mid b)$.

Definition. A natural number n is defined to be even if $2 \mid n$. And n is defined to be odd if $2 \nmid n$.

Problem 4 (10 pts) *Provide a proof of the following proposition:*

$$\forall n \in \mathbb{N}.\ (n \textit{ is even}) \vee (n \textit{ is odd})$$

Is your proof constructive? Why or why not?

Definition. A natural number a is composite if there exist natural numbers $n, k > 1$ such that $a = n \times k$. A natural number is prime if it is not composite.

Problem 5 (10 pts) *Provide an informal constructive proof of the fact that there are infinitely many prime numbers, if one exists.*

4 Quantifiers and Constructive Logic [20 pts]

Problem 6 (20 pts) *Let $P(n)$ be an arbitrary predicate on natural numbers (think of this predicate as a mathematical function that takes a number as input and returns a boolean). Consider the proposition:*

$$(\forall n \in \mathbb{N}.\ P(n) \vee \neg P(n)) \supset (\exists n. P(n) \vee \forall n. \neg P(n))$$

(i) Is this proposition true in classical logic? Justify your answer.

(ii) Is this proposition derivable in constructive logic? If it is, provide an informal derivation. If not, provide a justification.

(iii) If one exists, give an example of a predicate P such that the left hand side of the implication is true but the right hand side is not.